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## SOME PROPERTIES OF LUMPED-FILTER CIRCUITS FOR TRAVELING-WAVE TUBES

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TABLE OF FIGURES OF THE  
4-ELEMENT BAND PASS CIRCUIT

$x_1$	$\theta$	$y_{pn}$	$y_g$	$\frac{z_{OT}}{L_2}$	$\frac{z_{OT}}{C_1}$	$\frac{K_{TO}}{L_2}$	$\frac{K_{TO}}{C_1}$	$\frac{K_{T11}}{L_2}$	$\frac{K_{T11}}{C_1}$	$\frac{K_{LO}}{L_2}$	$\frac{K_{L11}}{C_1}$
10	7.2 -69-	7.3 -70-	7.4 -71-	7.5 -72-	7.6 -73-	7.7 -74-	7.8 -75-	7.9 -76-	7.10 -77-		
5	7.11 -78-	7.12 -79-	7.13 -80-	7.14 -81-	7.15 -82-	7.16 -83-	7.17 -84-	7.18 -85-	7.19 -86-		
2	7.20 -87-	7.21 -88-	7.22 -89-	7.23 -90-	7.24 -91-	7.25 -92-	7.26 -93-	7.27 -94-	7.28 -95-		
1.70	7.29 -96-	7.30 -97-	7.31 -98-	7.32 -99-	7.33 -100-	7.34 -101-	7.35 -102-	7.36 -103-	7.37 -104-		
1.6	7.38 -105-	7.39 -106-	7.40 -107-	7.41 -108-	7.42 -109-	7.43 -110-	7.44 -111-	7.45 -112-	7.46 -113-		
1.5	7.47 -114-	7.48 -115-	7.49 -116-	7.50 -117-	7.51 -118-	7.52 -119-	7.53 -120-	7.54 -121-	7.55 -122-		
0.5	7.56 -123-	7.57 -124-	7.58 -125-	7.59 -126-	7.60 -127-	7.61 -128-	7.62 -129-	7.63 -130-	7.64 -131-		
0.3	7.65 -132-	7.66 -133-	7.67 -134-	7.68 -135-	7.69 -136-	7.70 -137-	7.71 -138-	7.72 -139-	7.73 -140-		
0.16	7.74 -141-	7.75 -142-	7.76 -143-	7.77 -144-	7.78 -145-	7.79 -146-	7.80 -147-	7.81 -148-	7.82 -149-		
0.100	7.83 -150-	7.84 -151-	7.85 -152-	7.86 -153-	7.87 -154-	7.88 -155-	7.89 -156-	7.90 -157-	7.91 -158-		

## LIST OF SYMBOLS

b	Pierce's velocity parameter $b = \frac{u_0 - v_{pn}}{v_{pn}}$
c	velocity of light $v_c$
$C_n$	Pierce's circuit-beam coupling parameter $C_n^3 = I_0/4V_0 K_n$
d	length of the gap
f	frequency
$f_c$	cut-off frequency
g	subscript for group velocity
$m_r$	$= 1/\sqrt{1 + 4C_1/C_2}$
$m_s$	$= 1/\sqrt{1 + 4L_2/L_1}$
n	index for the nth space-harmonic component
r	capacitance ratio $r = 4C_1/C_2$
s	inductance ratio $s = 4L_2/L_1$
$u_0$	d-c velocity of the electron beam
$v_{pn}$	phase velocity of the nth space harmonic
$v_g$	group velocity
x	normalized frequency $x = f/f_c$
$x_1$	one of the normalized cut-off frequencies
$y_{pn}$	normalized phase velocity $y_{pn} = 2x/\beta_n D$
$y_g$	normalized group velocity $y_g = 2dx/d\theta$
$C_1$	capacitance in the $X_1$ arm
$C_2$	capacitance in the $X_2$ arm
D	length of one space-harmonic period
$E_n$	magnitude of the electric field of the nth space-harmonic component
$K_{Tn}$	interaction impedance of the nth space-harmonic component of the transverse interaction structure
$K_{Ln}$	interaction impedance of the nth space-harmonic component of the longitudinal interaction structure
$L_1$	inductance in the $X_1$ arm

## LIST OF SYMBOLS (Cont'd)

$L_2$	inductance in the $X_2$ arm
$M_n$	amplitude of the nth space-harmonic field component
P	total power carried by the circuit
V	peak voltage between line and ground
$V_0$	d-c beam voltage
$V_n$	synchronism voltage of the nth space harmonic component
$V_0$	$V_0 = V_n (1 + bC_n)^2$
$V_n$	$V_n = 2800 [y_{pn} f_c (\text{Mc/sec}) D (\text{cm})]^2 \text{ volts}$
	$= 18100 [y_{pn} f_c (\text{kMc/sec}) D (\text{inch})]^2 \text{ volts}$
$X_1$	reactance in arm 1 or the series arm
$X_2$	reactance in arm 2 or the shunt arm
$Z_{OT}$	characteristic circuit impedance of a T filter section
$Z_{OT}$	characteristic circuit impedance of a Pi filter section
$\beta_n D$	wave number of the nth space harmonic component $\beta_n D = \theta + 2\pi n$
$\theta$	circuit phase constant
$\omega$	angular frequency $\omega = 2\pi f$

#### I. INTRODUCTION

Since the development of the external-circuit traveling-wave tube, many different types of slow-wave circuits can be used with a traveling-wave tube structure with a single basic interaction geometry containing the electron beam and the interaction electrodes. To use such a versatile device it becomes necessary to investigate theoretically the properties of the various possible external circuits. Some sample calculations were obtained for the various types of circuits, either lumped or distributed-lumped, which can be used as slow-wave structures.<sup>1</sup>

In this report, results of more detailed calculations for several lumped-filter circuits are presented in normalized graphic form. The circuits include the low-pass, the high-pass,\* the three-element inductive band-pass, the three-element capacitive band-pass and the four-element band-pass circuits.

The characteristics presented include the phase-frequency characteristics, the phase velocity characteristics, the group velocity characteristics, the circuit impedance characteristics and the interaction impedance characteristics.

The interaction geometry considered here is uniformly periodic, i.e. all cylinders have equal lengths and all gaps have equal lengths. The quantities investigated are all normalized and plotted as a function of the normalized frequency.

\* These circuits can never be obtained physically in their exact forms due to distributed coil capacitance and lead inductance.

## II. GENERAL EQUATIONS

In this section a list of formulae for Tee and Pi filter circuits shown in Fig. 2.1 is given in terms of the reactances  $X_1$  and  $X_2$  and of the normalized frequency  $x$ . The circuit properties are

$$\text{phase constant } \theta = \pm 2 \sin^{-1} \sqrt{-\frac{X_1}{4X_2}} \quad (2.1)$$

phase velocity

$$v_{pn} = \frac{2x}{\theta + 2\pi f} \quad (2.2)$$

group velocity

$$v_g = 2 \frac{dx}{d\theta} \quad (2.3)$$

circuit impedances

$$Z_{0T} = \frac{\sqrt{X_1 X_2}}{\sqrt{1 + \frac{X_1}{4X_2}}} \quad (2.4)$$

$$Z_{0T} = \sqrt{X_1 X_2} \sqrt{1 + \frac{X_1}{4X_2}} \quad (2.5)$$

The interaction impedance is defined by

$$K_n = \frac{M_n^2 v^2}{\beta_n^2 D^2 Z_{0T}} \quad (2.6)$$

where the amplitude factor of the  $n$ th space harmonics is<sup>1</sup>

$$M_n^2 = 4 \sin^2 \frac{\beta_n D}{4} \quad \text{transversely loaded} \quad (2.7)$$

$$M_n^2 = 4 \sin^2 \frac{\beta_n D}{2} \quad \text{longitudinally loaded} \quad (2.8)$$

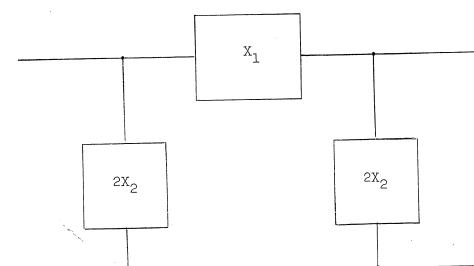
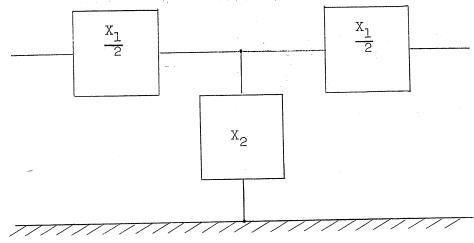


FIG. 2.1.--Tee and Pi filter sections.

Thus the longitudinal interaction impedance becomes

$$K_{Ln} = Z_{0\pi} \left( \frac{\sin \frac{\beta_n D}{2}}{\frac{\beta_n D}{2}} \right)^2 \quad (2.9)$$

and the transverse interaction impedance becomes

$$K_{Tn} = Z_{0\pi} \frac{1}{4} \left( \frac{\sin \frac{\beta_n D}{4}}{\frac{\beta_n D}{4}} \right)^2 \quad (2.10)$$

### III. CHARACTERISTICS OF THE LOW-PASS CIRCUITS

The low-pass circuits are shown in Fig. 3.1. The first two have transverse interaction gaps and the third one has longitudinal interaction gaps.

The equations for the characteristics are

$$\theta = 2 \sin^{-1} \frac{mx}{\sqrt{1-(1-m^2)x^2}} \quad (3.1)$$

$$y_{pn} = \frac{2x}{\theta + 2m\pi} \quad (3.2)$$

$$y_g = \frac{\sqrt{1-x^2}}{m} [1-(1-m^2)x^2] \quad (3.3)$$

$$z_{0\pi} = \sqrt{\frac{L_1}{C_2}} \left. \frac{1-x^2(1-m^2)}{\sqrt{1-x^2}} \right\} \quad (3.4)$$

$$z_{0t} = \sqrt{\frac{L_1}{C_2}} \left. \frac{\sqrt{1-x^2}}{1-x^2(1-m^2)} \right\} \quad (3.5)$$

$$z_{0t} = \sqrt{\frac{L_1}{C_2}} \left. \frac{\sqrt{1-x^2}}{1-x^2(1-m^2)} \right\} \quad (3.6)$$

$$z_{0\pi} = \sqrt{\frac{L_1}{C_2}} \left. \frac{1}{\sqrt{1-x^2}} \right\} \quad (3.7)$$

$$K_{Tn} = \sqrt{\frac{L_1}{C_2}} \left. \frac{1}{4} \left( \frac{\sin \frac{\beta_n D}{4}}{\frac{\beta_n D}{4}} \right)^2 \frac{1}{\sqrt{1-x^2} [1-x^2(1-m^2)]} \right\} \quad (3.8)$$

Fig. 2(a)

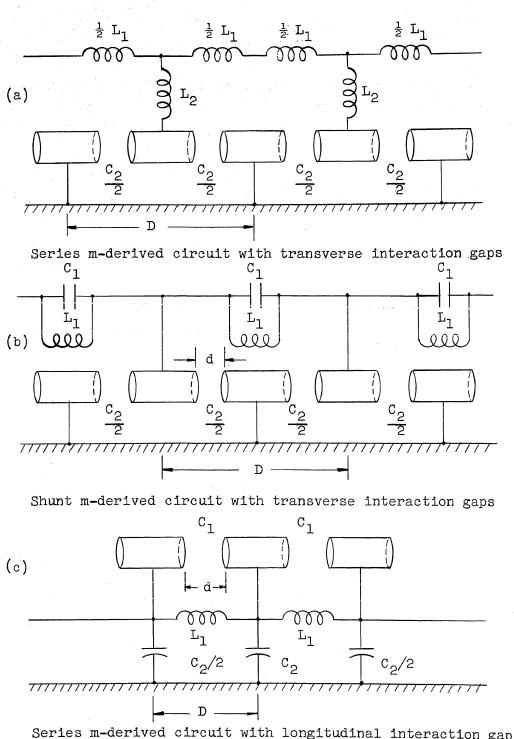


FIG. 3.1. - M-derived low-pass circuits.

$$K_{Tn} = \sqrt{\frac{L_1}{C_2}} \cdot \frac{1}{4} \left( \frac{\sin \frac{\beta_n D}{4}}{\frac{\beta_n D}{4}} \right)^2 \cdot \frac{1}{\sqrt{1-x^2}} \quad \text{Fig. 2(b)} \quad (3.9)$$

$$K_{Ln} = \sqrt{\frac{L_1}{C_2}} \left( \frac{\sin \frac{\beta_n D}{2}}{\frac{\beta_n D}{2}} \right)^2 \cdot \frac{1}{\sqrt{1-x^2}} \quad \text{Fig. 2(c)} \quad (3.10)$$

These equations are plotted as curves in Figs. 3.2 to 3.12.

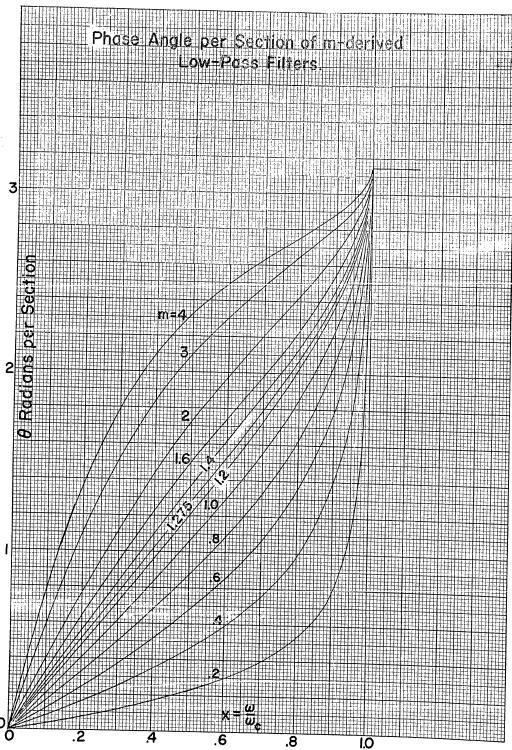


FIG. 3.2--Phase angle per section of low-pass circuits.

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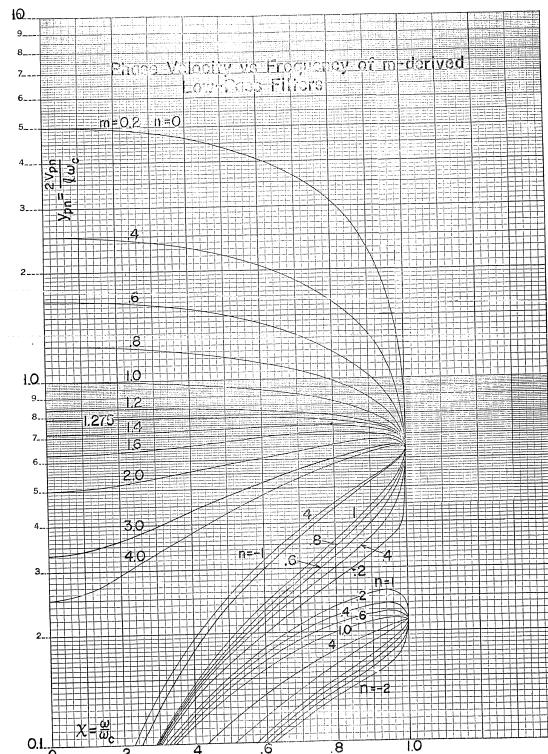


FIG. 3.3--Phase velocity of low-pass circuits.

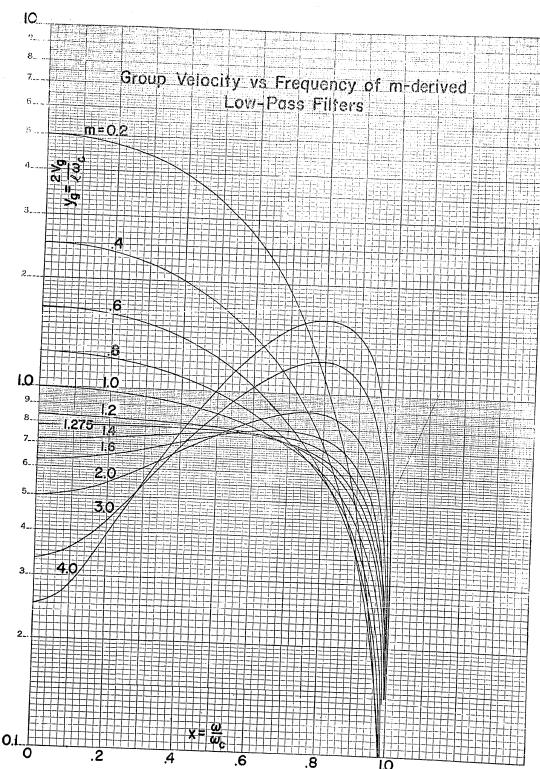


FIG. 3.4.--Group velocity of low-pass circuits.

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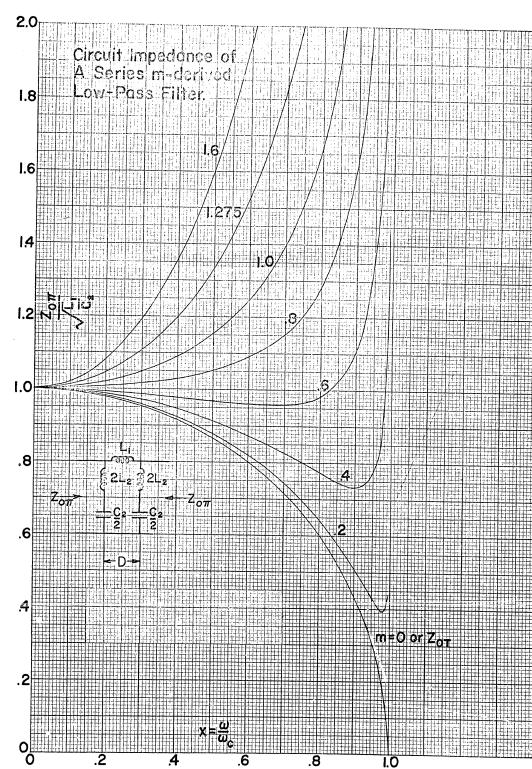


FIG. 3.5.--Circuit impedance of low-pass circuits of Pi section.

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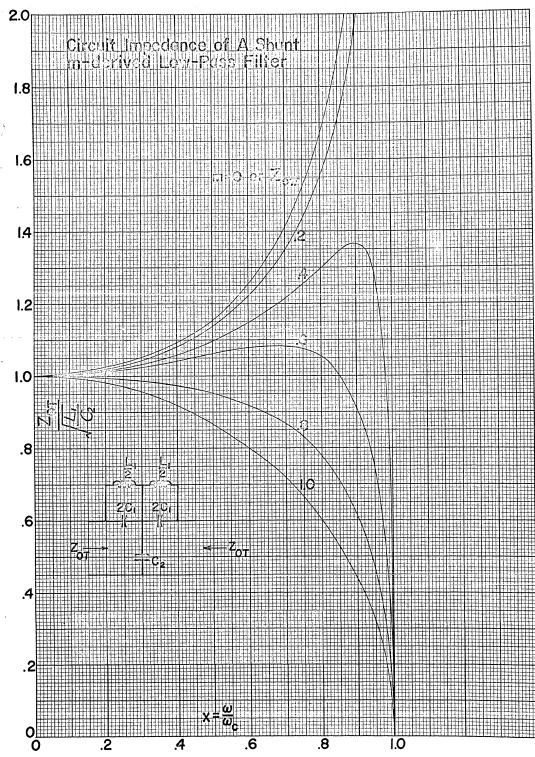


FIG. 3.6.--Circuit impedance of low-pass circuits of Tee section.

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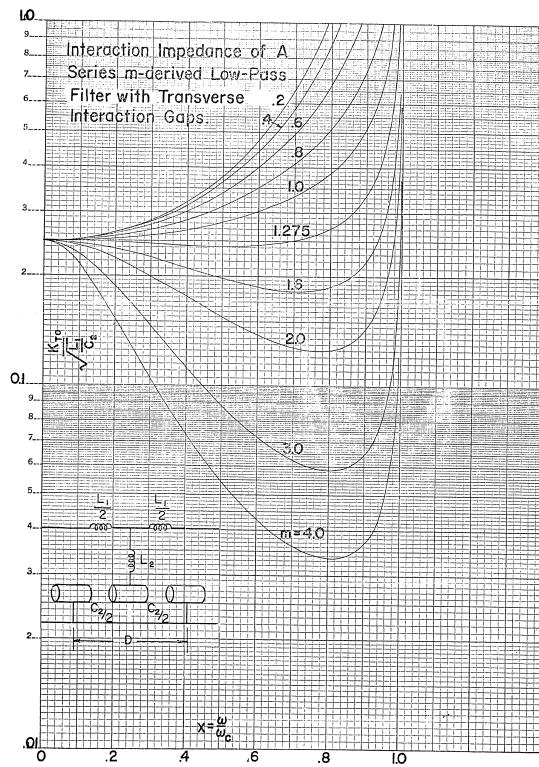


FIG. 3.7.--Transverse interaction impedance  $K_{T0}$  of a series  $m$ -derived low-pass circuit.

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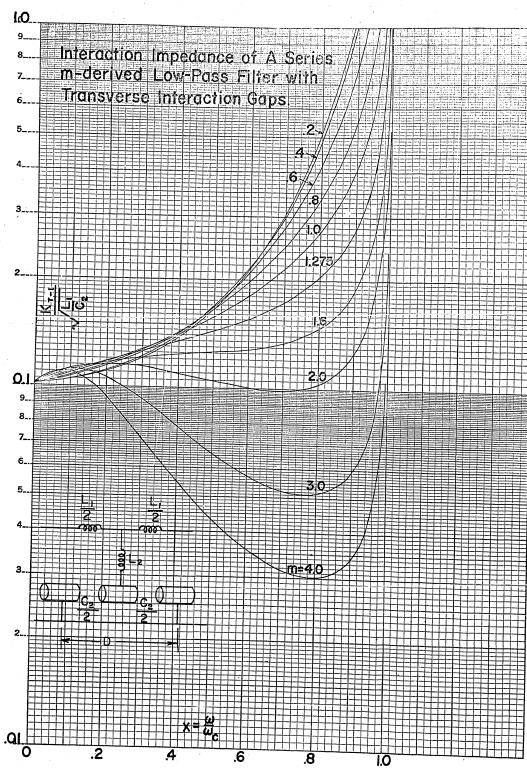
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FIG. 3.8.--Transverse interaction impedance  $K_{T-1}$  of a series  $m$ -derived low-pass circuit.

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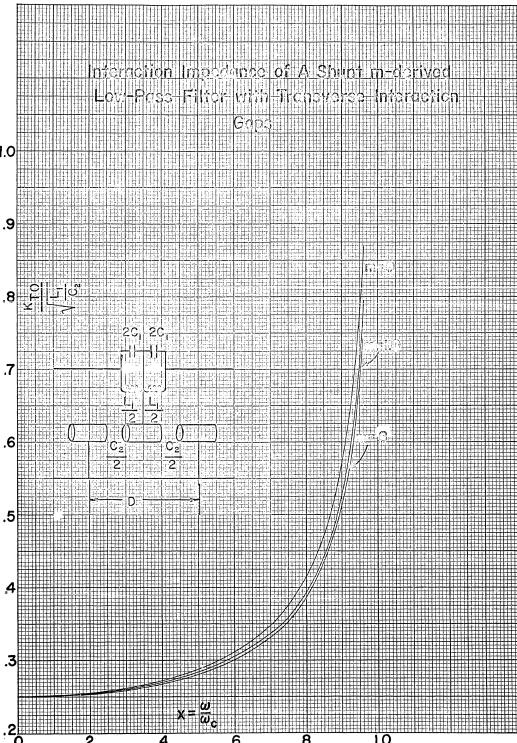


FIG. 3.9.--Transverse interaction impedance  $K_{TO}$  of a shunt  $m$ -derived low-pass circuit.

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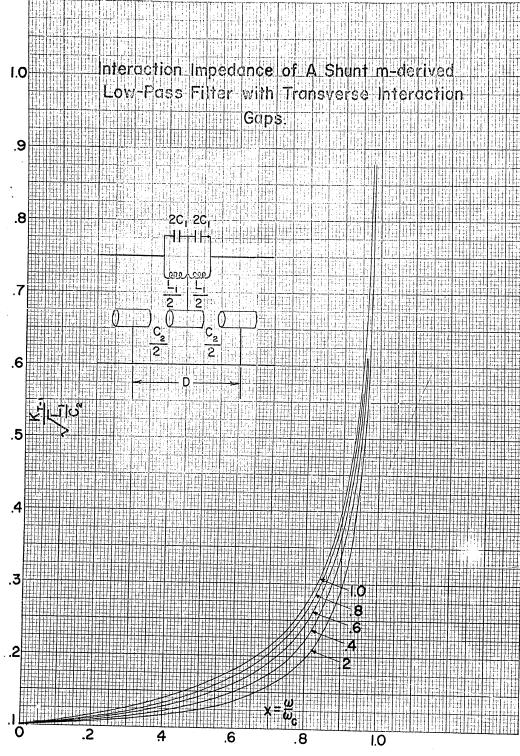


FIG. 3.10.--Transverse interaction impedance  $K_{t-10}$  of a shunt  $m$ -derived low-pass circuit.

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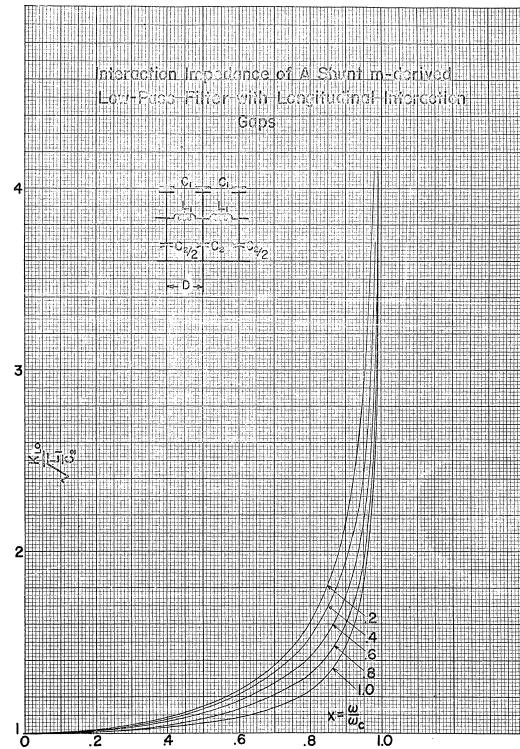
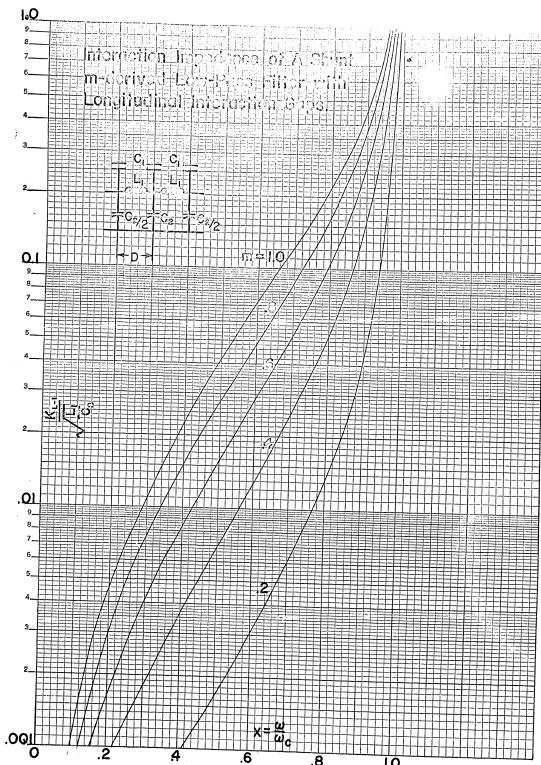


FIG. 3.11.--Longitudinal interaction impedance  $K_{L0}$  of a shunt  $m$ -derived low-pass circuit.

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## IV. CHARACTERISTICS OF THE HIGH-PASS CIRCUITS.

The high-pass circuits are shown in Fig. 4.1. The first two have longitudinal interaction gaps and the third one has transverse interaction gaps.

The equations for the characteristics are

$$\theta = -2\sin^{-1} \frac{m}{\sqrt{x^2-1+m^2}} \quad (4.1)$$

$$|y_{pn}| = \frac{2x}{|\theta+2n\pi|} \quad (4.2)$$

$$y_g = \frac{\sqrt{x^2-1}}{mx} \quad (x^2-1+m^2) \quad (4.3)$$

$$z_{0\pi} = \sqrt{\frac{L_2}{C_1}} \quad \left. \begin{array}{l} \frac{x^2-1+m^2}{x \sqrt{x^2-1}} \\ \end{array} \right\} \quad (4.4)$$

$$z_{0T} = \sqrt{\frac{L_2}{C_1}} \quad \left. \begin{array}{l} \frac{\sqrt{x^2-1}}{x} \\ \end{array} \right\} \quad (4.5)$$

$$z_{0T} = \sqrt{\frac{L_2}{C_1}} \quad \left. \begin{array}{l} \frac{x}{x^2-1+m^2} \\ \end{array} \right\} \quad (4.6)$$

$$z_{0\pi} = \sqrt{\frac{L_2}{C_1}} \quad \left. \begin{array}{l} \frac{x}{\sqrt{x^2-1}} \\ \end{array} \right\} \quad (4.7)$$

$$K_{Ln} = \sqrt{\frac{L_2}{C_1}} \left( \frac{\sin \frac{B_n D}{2}}{\frac{B_n D}{2}} \right)^2 \quad \left. \begin{array}{l} \frac{x^2-1+m^2}{x \sqrt{x^2-1}} \\ \end{array} \right\} \quad (4.8)$$

Fig. 14(a) and (c)

Fig. 14(b)

Fig. 14(a)

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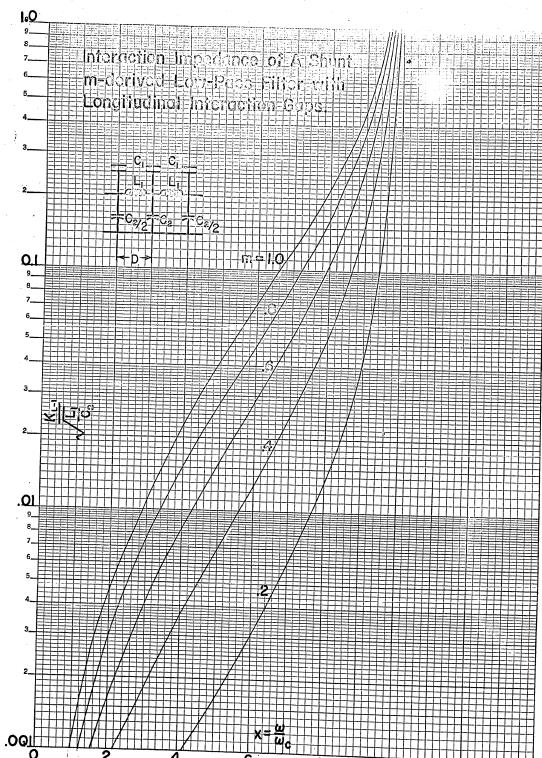


FIG. 3.12.--Longitudinal interaction impedance  $K_{L-1}$  of a shunt m-derived low-pass circuit

#### IV. CHARACTERISTICS OF THE HIGH-PASS CIRCUITS.

The high-pass circuits are shown in Fig. 4.1. The first two have longitudinal interaction gaps and the third one has transverse interaction gaps.

The equations for the characteristics are

$$\theta = -2 \sin^{-1} \frac{m}{\sqrt{x^2 - 1 + m^2}} \quad (4.1)$$

$$|y_{pn}| = \frac{2x}{|\theta + 2n\pi|} \quad (4.2)$$

$$y_g = \frac{\sqrt{x^2 - 1}}{mx} \quad (x^2 - 1 + m^2) \quad (4.3)$$

$$= \lambda_1 - \lambda_2 + (\lambda_3 - \lambda_4) \lambda_1$$

$$z_{\text{opt}} = \sqrt{\frac{L_2}{C}} \quad \left. \begin{array}{l} x \\ \frac{\sqrt{x^2 - 1}}{2} \end{array} \right\} \quad (4.6)$$

Fig. 14(b)

$$z_{0\pi} = \sqrt{\frac{L_2}{C_1}} \cdot \frac{x}{\sqrt{x^2 - 1}} \quad ] \quad (4.7)$$

Fig. 14(b) (4.7)

$$K_{Ln} = \sqrt{\frac{L_2}{C_1}} \left( \frac{\sin \frac{B_n D}{2}}{\frac{B_n D}{2}} \right)^2 \frac{x^2 - 1 + m^2}{x \sqrt{x^2 - 1}} \quad \text{Fig. 14(a)} \quad (4.8)$$

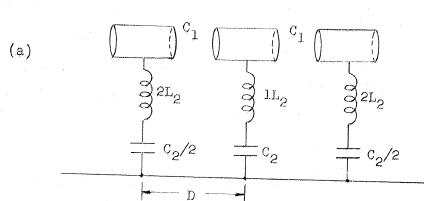
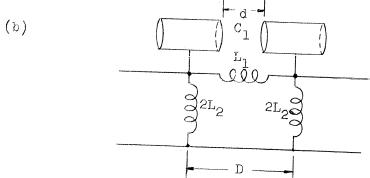
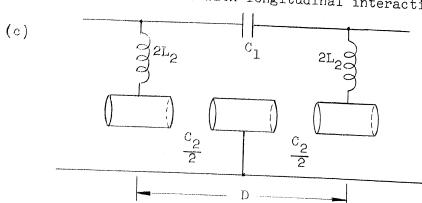
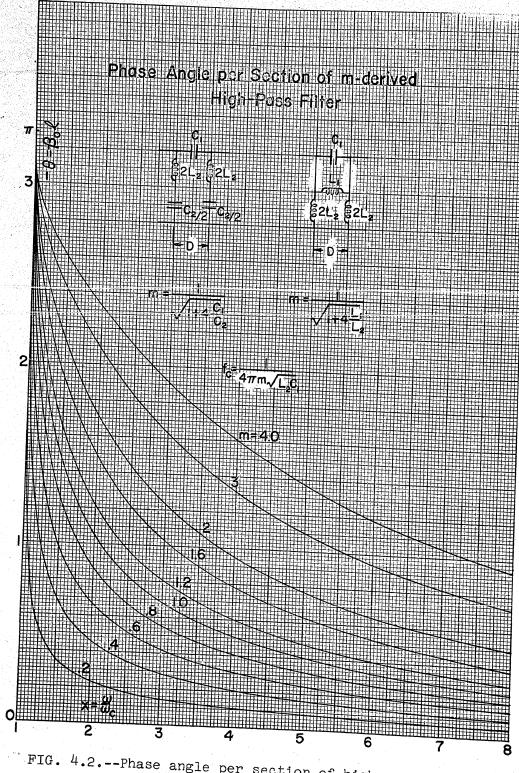
Series  $m$ -derived circuit with longitudinal interaction gaps.Shunt  $m$ -derived circuit with longitudinal interaction gaps.Series  $m$ -derived circuit with transverse interaction gaps.

FIG. 4.1--High-pass circuits.

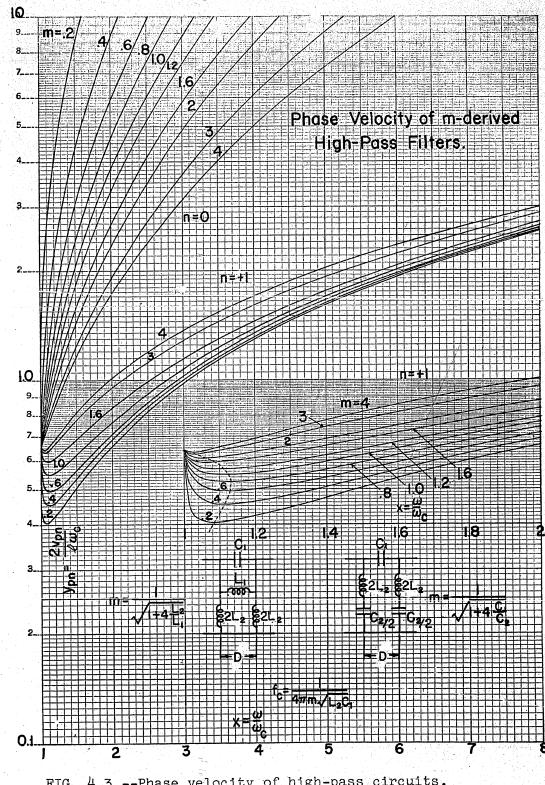
$$K_{Ln} = \sqrt{\frac{L_2}{C_1}} \left( \frac{\sin \frac{\beta_n D}{2}}{\frac{\beta_n D}{2}} \right)^2 \frac{x}{\sqrt{x^2 - 1}} \quad \text{Fig. 14(b)} \quad (4.9)$$

$$K_{Tn} = \sqrt{\frac{L_2}{C_1}} \left( \frac{\sin \frac{\beta_n D}{4}}{\frac{\beta_n D}{4}} \right)^2 \frac{x^2 - 1 + m^2}{x \sqrt{x^2 - 1}} \quad \text{Fig. 14(c)} \quad (4.10)$$

These equations are plotted as curves in Figs. 4.2 to 4.12.



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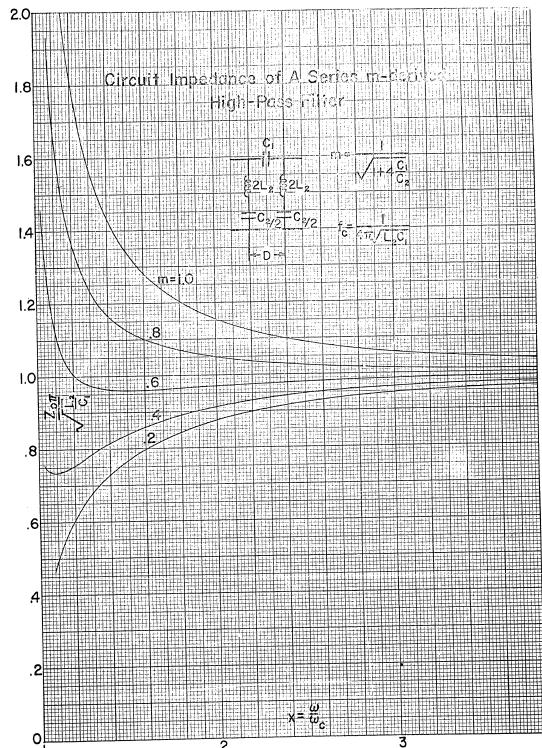
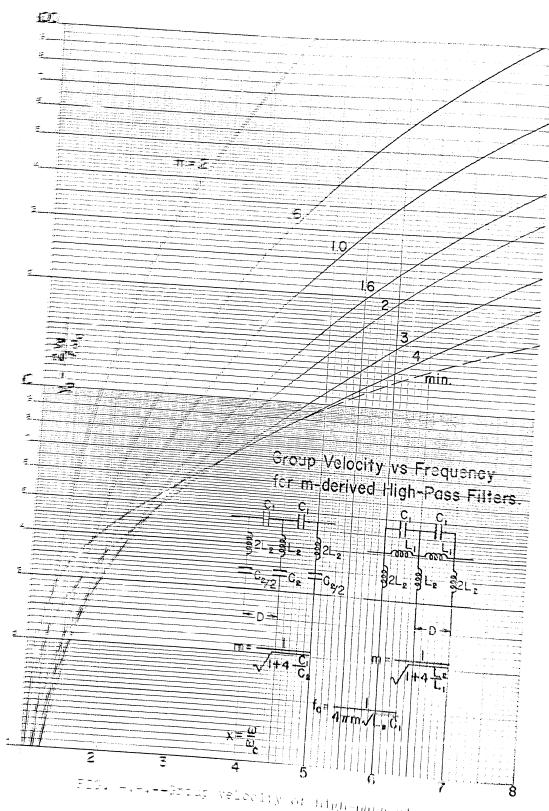


FIG. 4.5.--Circuit impedance of high-pass circuits of Pi section.

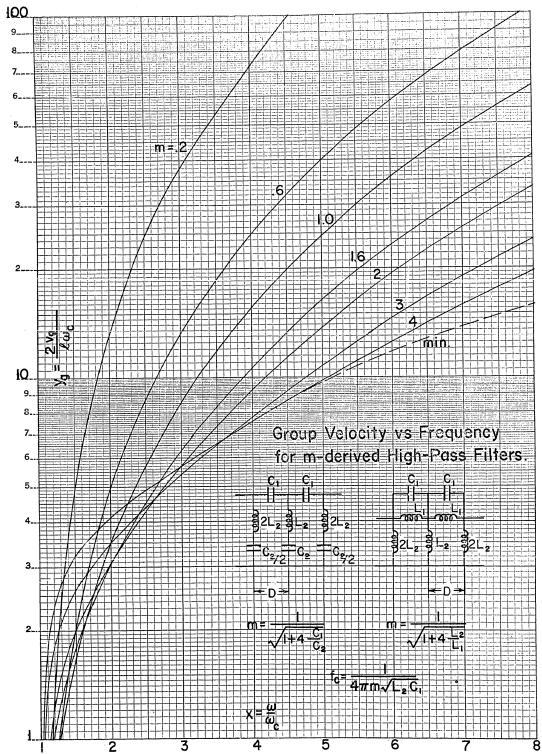


FIG. 4.4.--Group velocity of high-pass circuits.

- 24 -

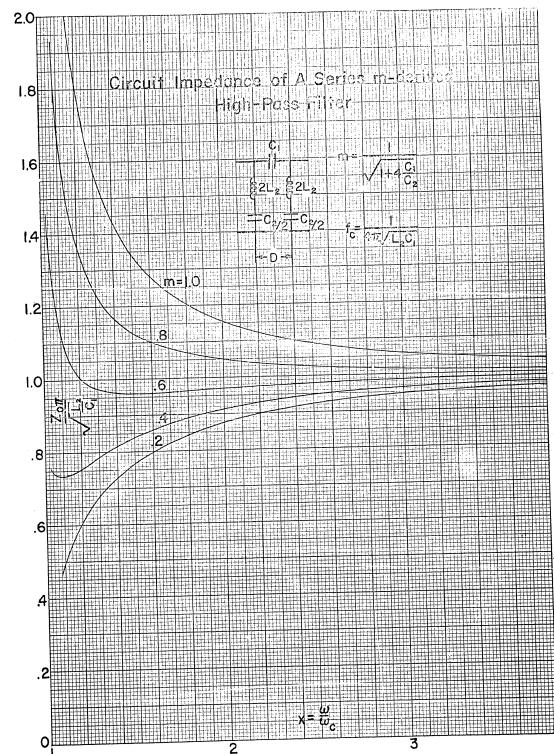


FIG. 4.5.--Circuit impedance of high-pass circuits of Pi section.

- 25 -

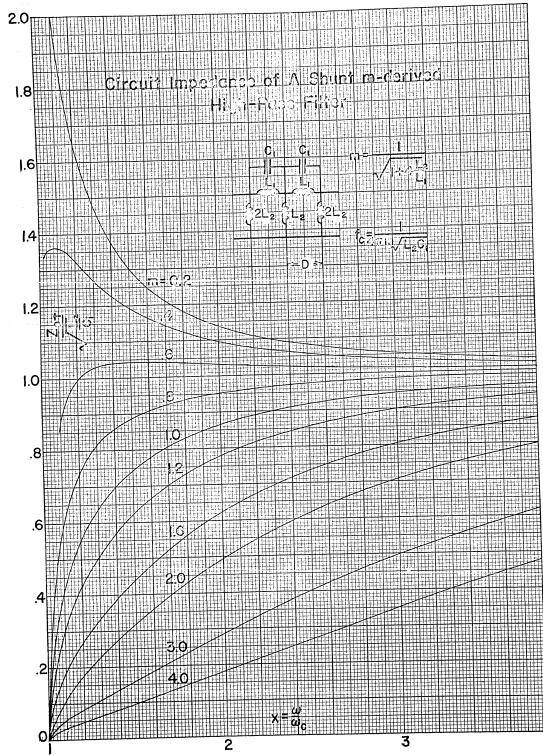


FIG. 4.6.--Circuit impedance of high-pass circuits of Tee section.

- 26 -

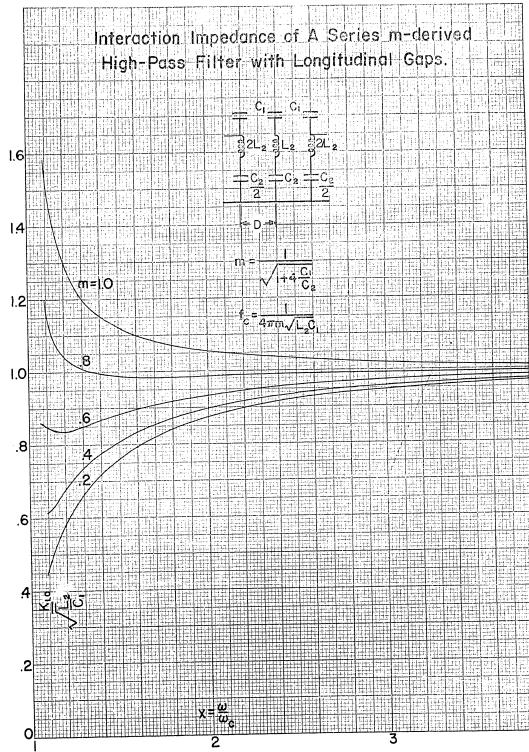


FIG. 4.7.--Longitudinal interaction impedance  $K_{L0}$  of a series m-derived high-pass circuit.

- 27 -

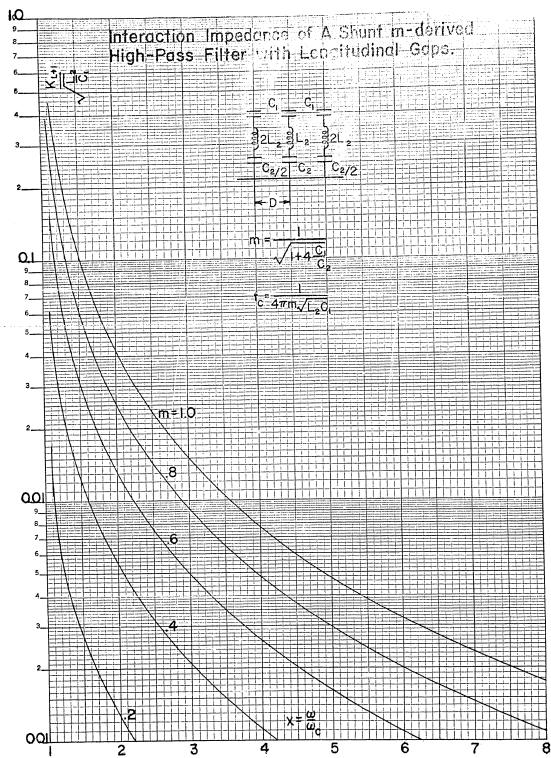


FIG. 4.8--Longitudinal interaction impedance  $K_{L+1}$  of a series  $m$ -derived high-pass circuit.

- 28 -

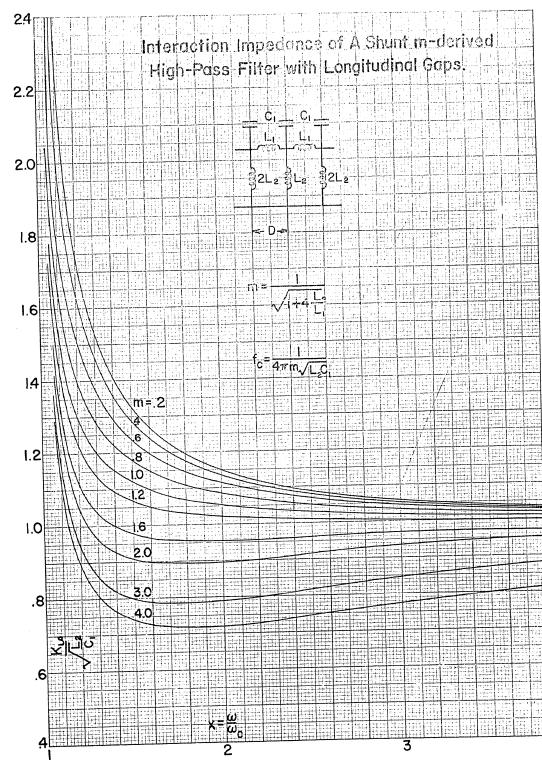


FIG. 4.9--Longitudinal interaction impedance  $K_{L0}$  of a shunt  $m$ -derived high-pass circuit.

- 29 -

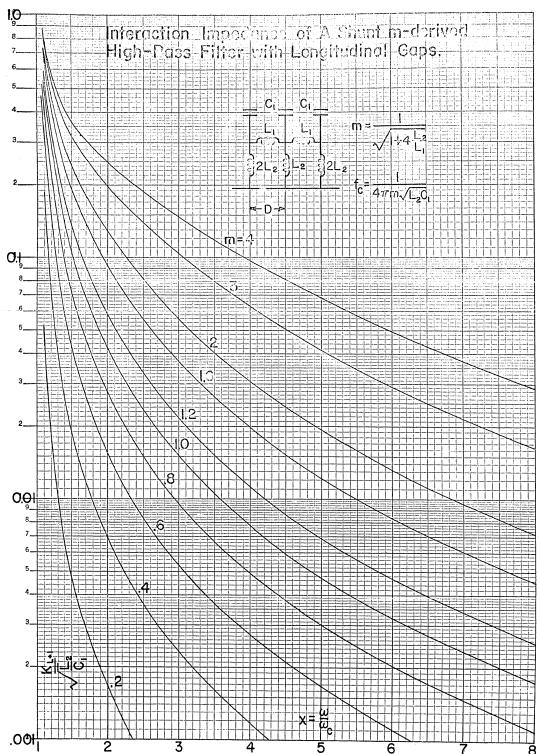


FIG. 4.10.--Longitudinal interaction impedance  $K_{L+1}$  of a shunt  $m$ -derived high-pass circuit.

- 30 -

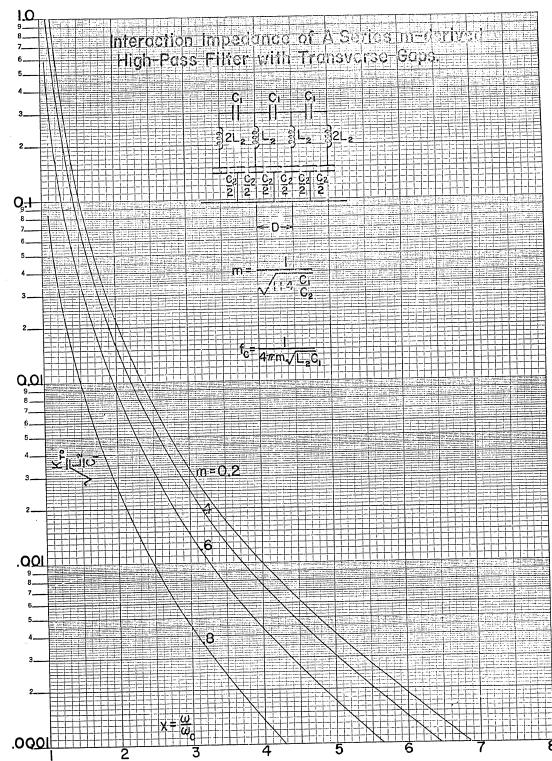
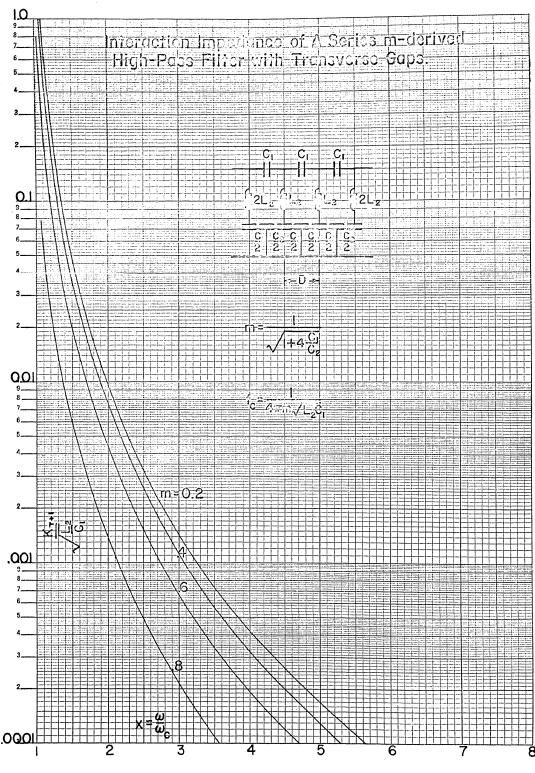


FIG. 4.11.--Transverse interaction impedance  $K_{T0}$  of a series  $m$ -derived high-pass circuit.

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#### V. CHARACTERISTICS OF THE THREE-ELEMENT INDUCTIVELY COUPLED BAND-PASS CIRCUIT

The inductively coupled band-pass circuit is shown in Fig. 5.1.

The equations for the characteristics are

$$\theta = \pm_2 \sin^{-1} \sqrt{\frac{x^2 - m^2}{1 - m^2}} \quad (5.1)$$

$$y_{pn} = \frac{2x}{\theta + 2\pi} \quad (5.2)$$

$$y_g = \frac{\sqrt{1-x^2}}{x} \sqrt{\frac{x^2 - m^2}{1 - m^2}} \quad (5.3)$$

$$z_{0\pi} = \sqrt{\frac{L_1}{C_2}} \frac{\sqrt{1-m^2}}{\sqrt{x^2-m^2}} \frac{x}{\sqrt{1-x^2}} \quad (5.4)$$

$$z_{0T} = \sqrt{\frac{L_1}{C_2}} \frac{x \sqrt{x^2-1}}{\sqrt{m^2-x^2} \sqrt{1-m^2}} \quad (5.5)$$

$$K_{Tn} = \left( \frac{\sin \frac{\beta_n D}{4}}{\frac{\beta_n D}{4}} \right)^2 \frac{1}{4} \sqrt{\frac{L_1}{C_2}} \frac{\sqrt{1-m^2}}{\sqrt{x^2-m^2}} \frac{x}{\sqrt{1-x^2}} \quad (5.6)$$

The cutoff frequencies are given by  $x_c = 1$  or  $\omega_c = 1/m_s \sqrt{L_2 C_2}$  and  $x_1 = m_s$  where  $m_s = 1/\sqrt{1+4L_2/L_1}$

For positive mutual coupling, i.e. when  $L_2$  and  $L_1$  are both greater than zero, the circuit is fundamentally a forward-wave circuit and the normalized pass-band is between  $m_s$  and 1.

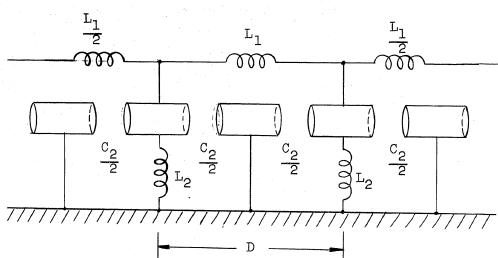


FIG. 5.1.--Three-element inductively coupled band-pass circuit.

When  $L_2$  or  $L_1$  is negative, the circuit is fundamentally a backward-wave circuit and the normalized pass-band is between 1 and  $m_s$ . For the fundamental forward-wave circuit, the positive sign is to be used in the  $\epsilon$  expression and for the fundamental backward-wave circuit the negative sign is to be used in the  $\epsilon$  expression. The negative-mutual-coupled circuit requires a center-tapped coil with the two halves wound in the same direction. The positive-mutual-coupled circuit on the other hand requires a center-tapped coil with the two halves wound in the opposite direction. A band-pass space-harmonic amplifier can be designed using the negative-mutual-coupled circuit with good bandwidth when the pass band is between about 1 to 1.60.

Equations (4.1) to (4.6) are plotted in Figs. 5.2 to 5.16. Cases for  $m_s = 1.5, 1.6$  and  $1.7$  are also plotted on an enlarged scale. Curves include both the positive and the negative-mutual-coupling cases.

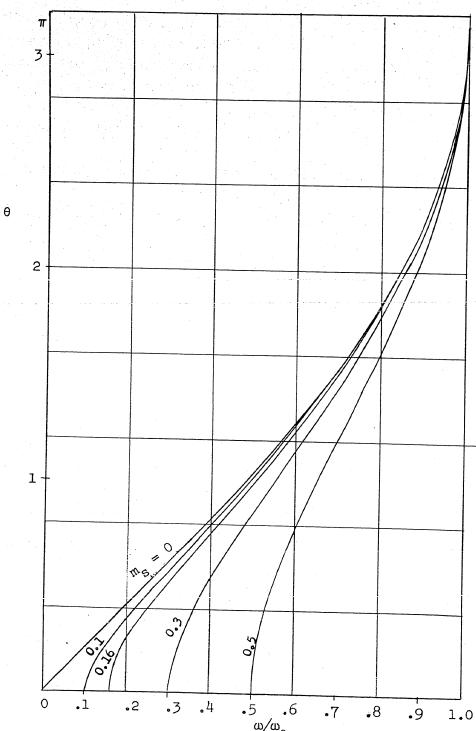


FIG. 5.2.--Phase angle per section of the positive mutual coupled three-element band-pass circuit.

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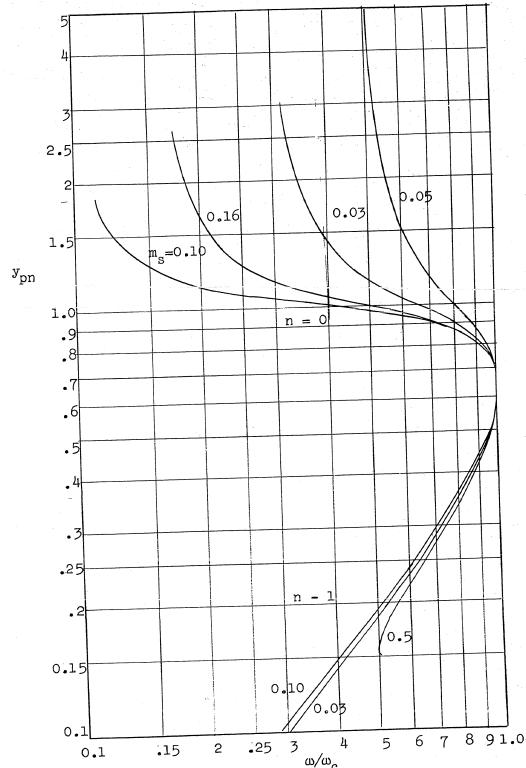


FIG. 5.3.--Phase velocity  $y_{pn}$  of the positive-mutual-coupled three-element band-pass circuit.

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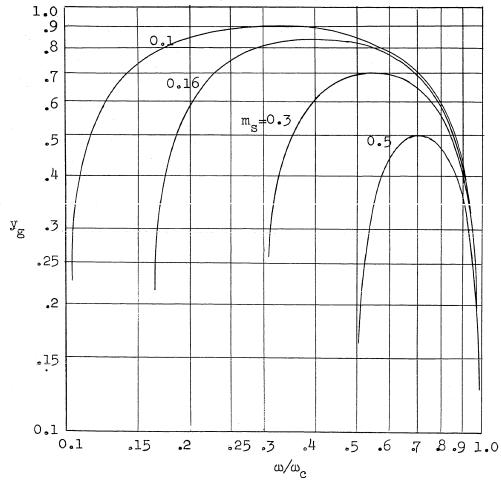


FIG. 5.4.--Group velocity  $y_g$  of the positive-mutual-coupled three-element band-pass circuit.

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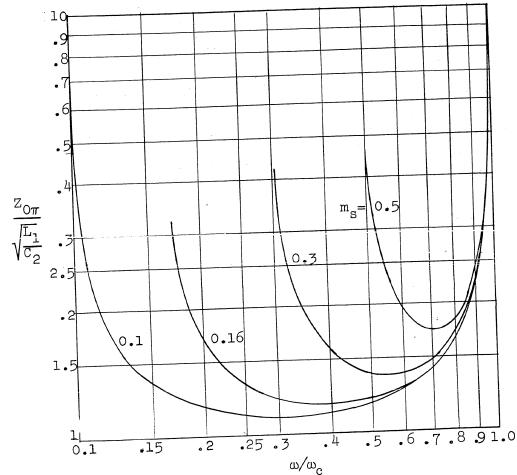


FIG. 5.5.--Circuit impedance  $Z_{0w}$  of the positive-mutual-coupled three-element band-pass circuit of PI section.

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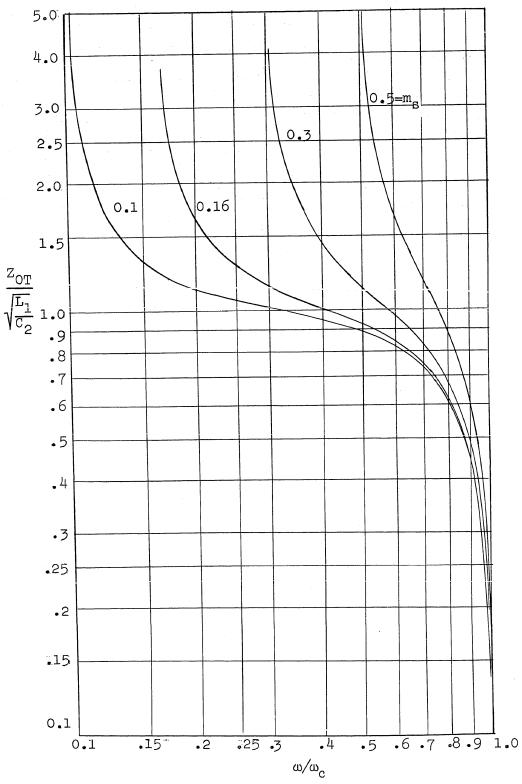


FIG. 5.6.--Circuit impedance  $Z_{OT}$  of the positive-mutual coupled three-element band-pass circuit of Tee section.

- 40 -

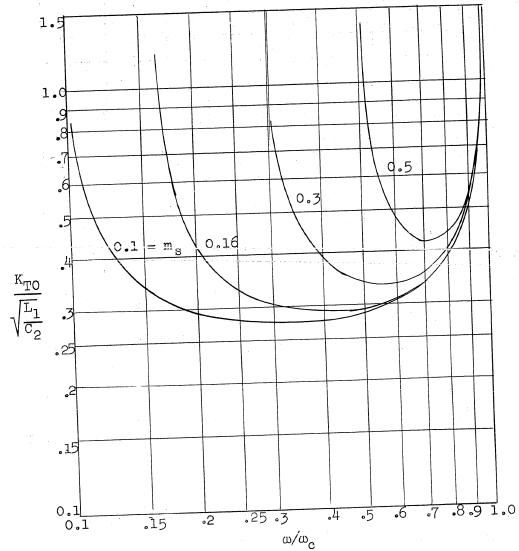


FIG. 5.7.--Transverse interaction impedance  $K_{TO}$  of the positive-mutual-coupled three-element band-pass circuit.

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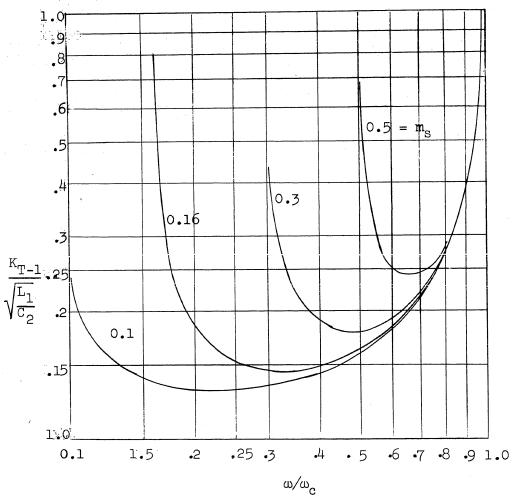


FIG. 5.8.--Transverse interaction impedance  $K_{T-1}$  of the positive-mutual-coupled three-element band-pass circuit.

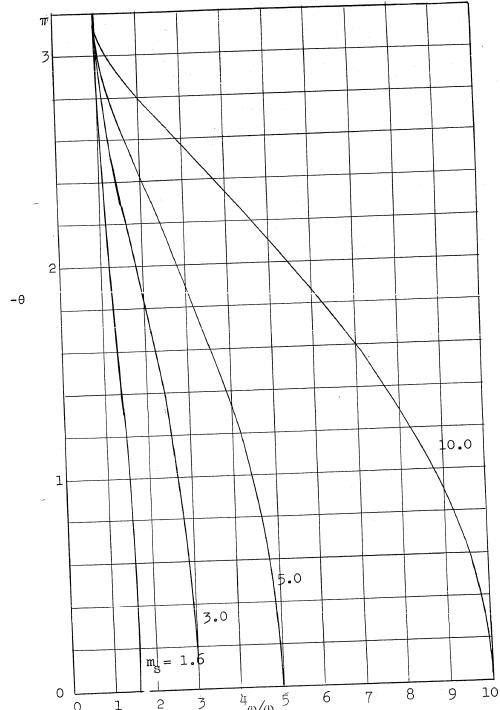


FIG. 5.9.--Phase angle per section of the negative-mutual-coupled three-element band-pass circuit.

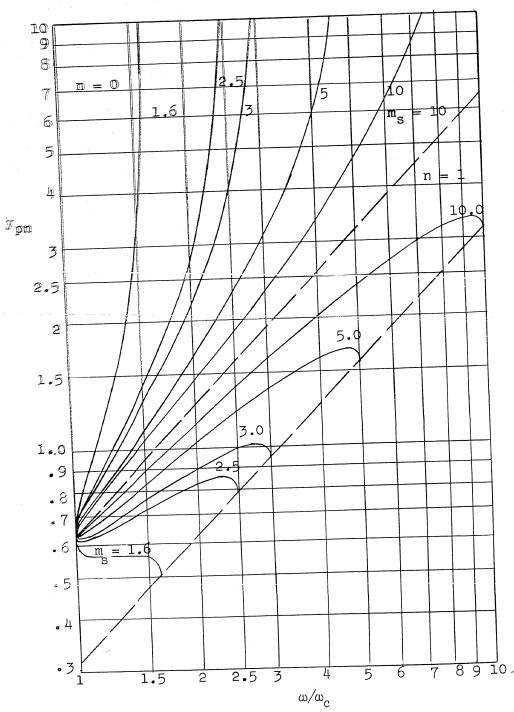


FIG. 5.10.--Phase velocity  $y_{BB}$  of the negative-mutual-coupled three-element band-pass circuit.

- 44 -

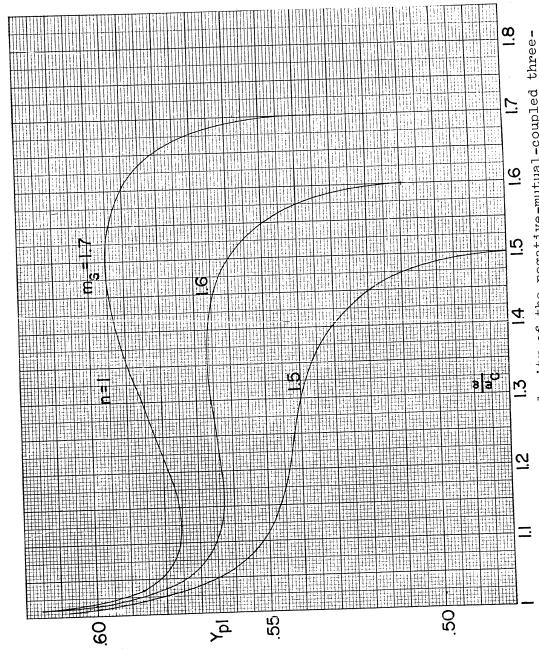


FIG. 5.11.--Phase velocity of the negative-mutual-coupled three-element band-pass circuit (enlarged scale).

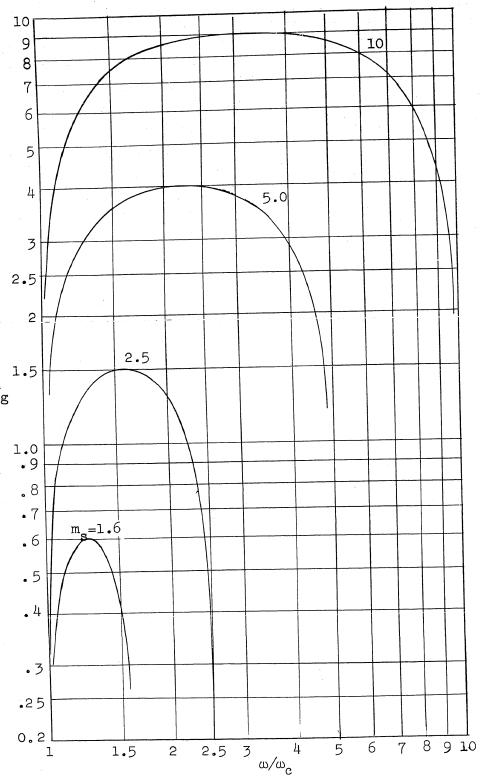


FIG. 5.12.--Group velocity  $y_g$  of the negative-mutual-coupled three-element band-pass circuit.

- 46 -

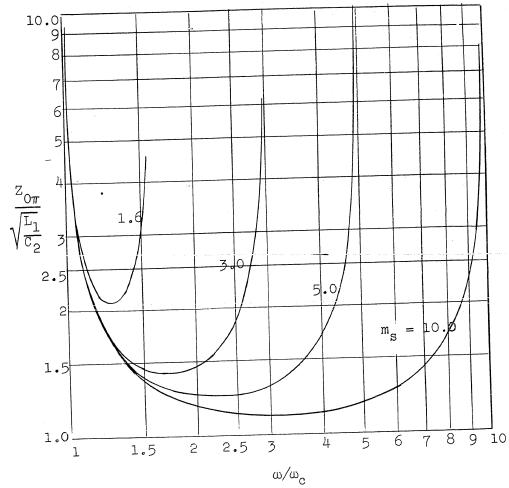


FIG. 5.13.--Circuit impedance  $Z_{0\pi}$  of the negative-mutual-coupled three-element band-pass circuit of Pi section.

- 47 -

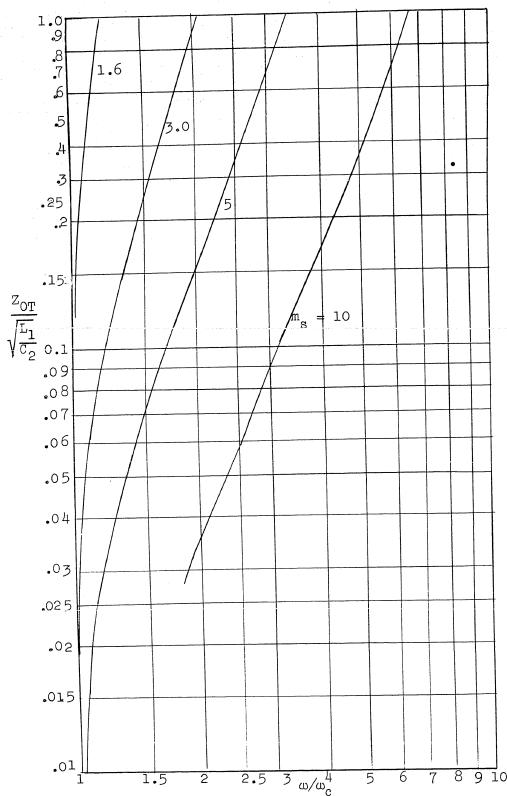


FIG. 5.14.--Circuit impedance  $Z_{OT}$  of the negative-mutual-coupled three-element band-pass circuit of Tee section.

- 48 -

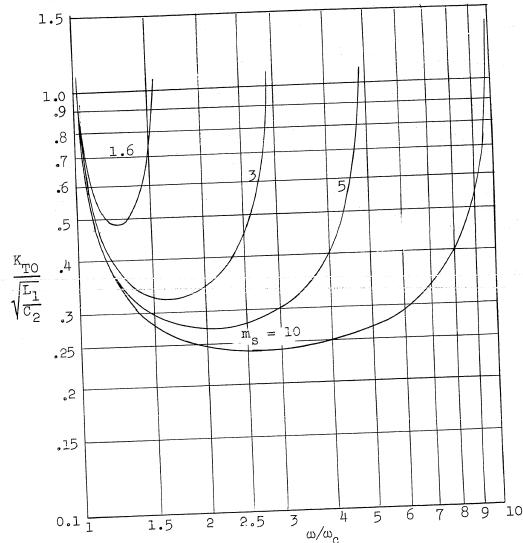


FIG. 5.15.--Transverse interaction impedance  $K_{TO}$  of the negative-mutual-coupled three-element band-pass circuit.

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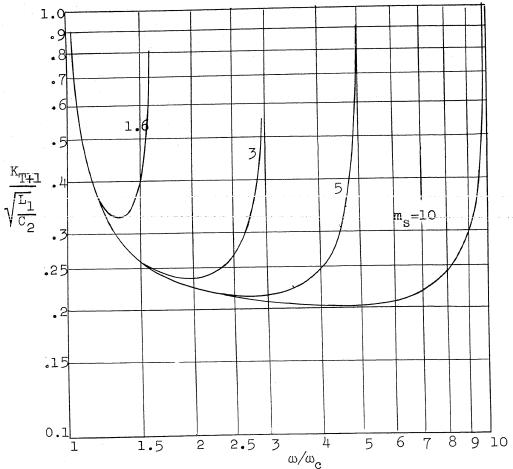


FIG. 5.16.--Transverse interaction impedance  $K_{Tn1}$  of the negative-mutual-coupled three-element band-pass circuit.

## VI. CHARACTERISTICS OF THE THREE-ELEMENT CAPACITIVELY COUPLED BAND-PASS CIRCUITS

The capacitively coupled band-pass circuits are shown in Figs. 6.1 (a) and (b). The equations for the characteristics are

$$\theta = -2\sin^{-1} \sqrt{\frac{x^2 - m_r^2}{1 - m_r^2}} \quad (6.1)$$

$$y_{pn} = \frac{2x}{|\theta + 2\pi|} \quad (6.2)$$

$$y_g = x^2 \sqrt{x^2 - 1} \sqrt{x^2 - m_r^2} \quad (6.3)$$

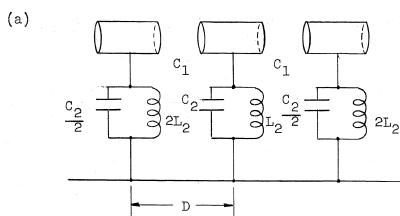
$$z_{0\pi} = \sqrt{\frac{L_2}{C_1}} \frac{\sqrt{1 - m_r^2}}{x \sqrt{1 - x^2} \sqrt{x^2 - m_r^2}} \quad (6.4)$$

$$z_{0t} = \sqrt{\frac{L_2}{C_1}} \frac{\sqrt{1 - x^2}}{\sqrt{1 - m_r^2} \sqrt{x^2 - m_r^2} x} \quad (6.5)$$

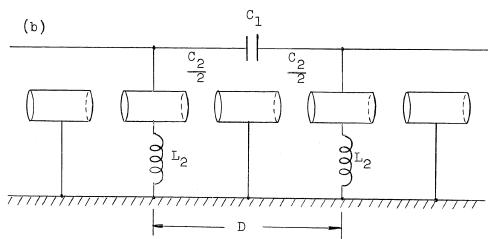
$$K_{Ln} = \sqrt{\frac{L_2}{C_1}} \frac{\sqrt{1 - m_r^2}}{x \sqrt{1 - x^2} \sqrt{x^2 - m_r^2}} \left( \frac{\sin \frac{\beta_n D}{2}}{\frac{\beta_n D}{2}} \right)^2 \quad (6.6)$$

$$K_{Tn} = \sqrt{\frac{L_2}{C_1}} \frac{\sqrt{1 - m_r^2}}{x \sqrt{1 - x^2} \sqrt{x^2 - m_r^2}} \left( \frac{\sin \frac{\beta_n D}{4}}{\frac{\beta_n D}{4}} \right)^2 \quad (6.7)$$

$$\text{and } \omega_c = \frac{m_r}{\sqrt{L_2 C_2}} \quad \text{where } m_r = x_1^{-1} = \sqrt{\frac{1}{1 + \frac{4C_1}{C_2}}}$$



Longitudinal interaction structure



Transverse interaction structure

FIG. 6.1.--Three-element capacitively coupled band-pass circuits.

The capacitively coupled circuit is a backward-wave circuit. There are two possible locations for the interaction electrodes, i.e., at  $C_1$  for the longitudinal type and at  $C_2$  for the transverse type.

Equations (6.1) to (6.7) are plotted in Figs. 6.2 to 6.13.

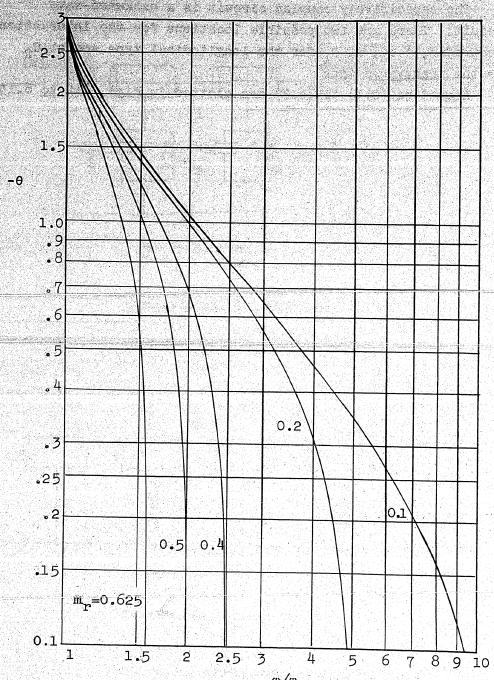


FIG. 6.2.--Phase angle per section of the three-element capacitively coupled band-pass circuit.

- 54 -

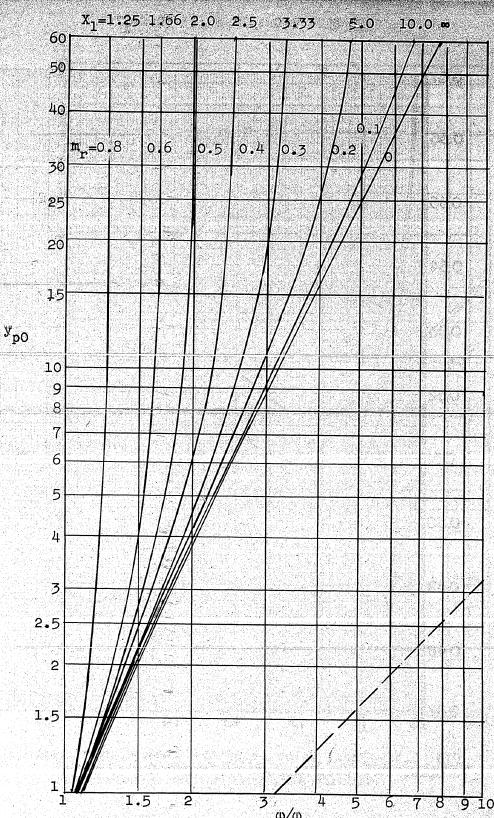


FIG. 6.3.--Phase velocity  $y_{p0}$  of the three-element capacitively coupled band-pass circuit.

- 55 -

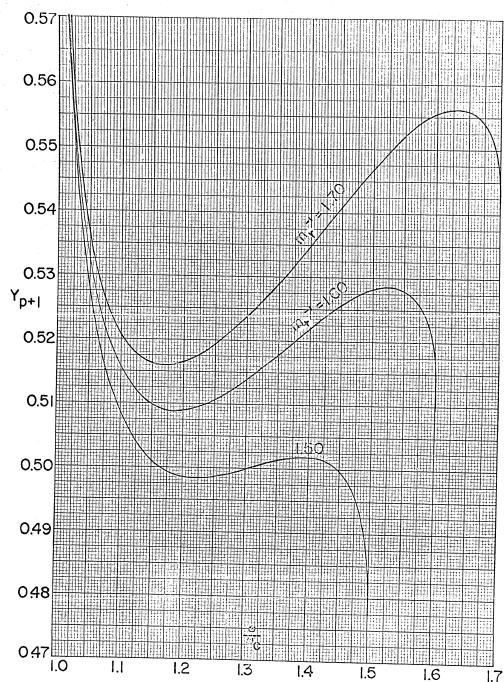


FIG. 6.4.--Phase velocity of the three-element capacitively coupled band-pass circuit (enlarged scale)  $y_{p+1}$ .

- 56 -

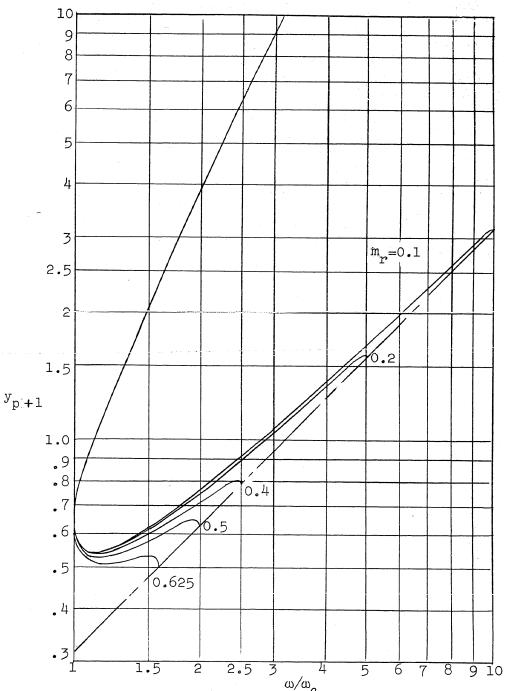


FIG. 6.5.--Phase velocity  $y_{p+1}$  of the three-element capacitively coupled band-pass circuit.

- 57 -

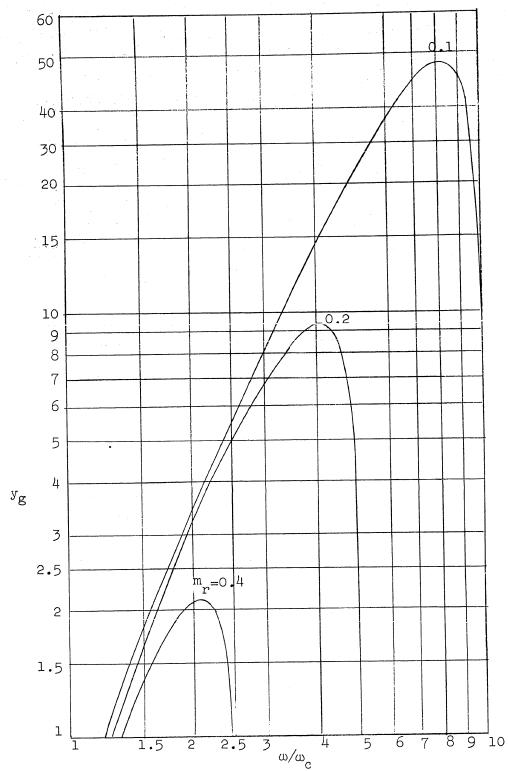


FIG. 6.6.--Group velocity  $y_g$  of the three-element capacitive coupled band-pass circuit.

- 58 -

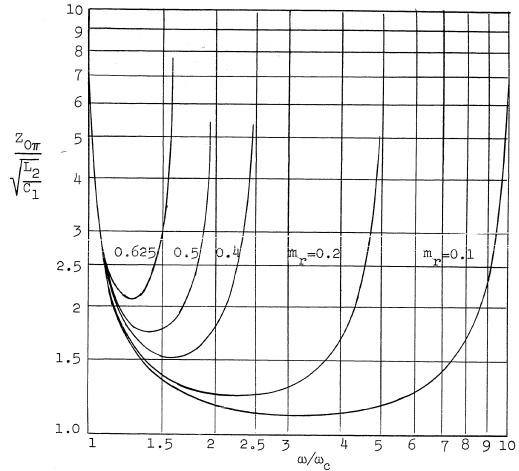


FIG. 6.7.--Circuit impedance  $Z_{0\pi}$  of the three element capacitive coupled band-pass circuit of Pi section.

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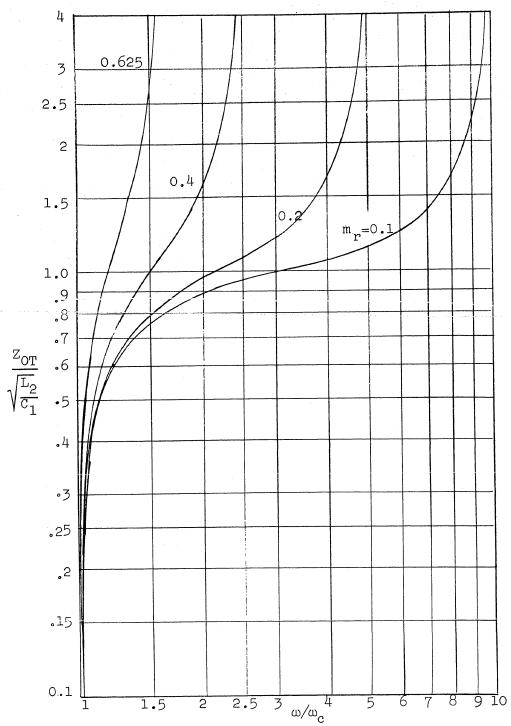


FIG. 6.8.--Circuit impedance  $Z_{OT}$  of the three-element capacitive coupled band-pass circuit of Tee section.

- 60 -

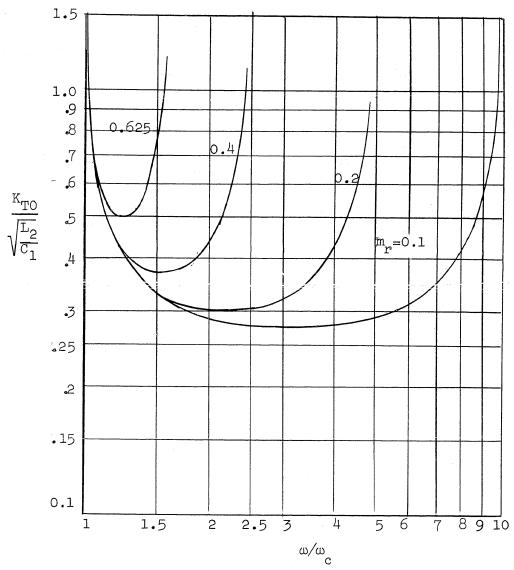


FIG. 6.9.--Transverse interaction impedance  $K_{TO}$  of the three-element capacitively-coupled band-pass circuit.

- 61 -

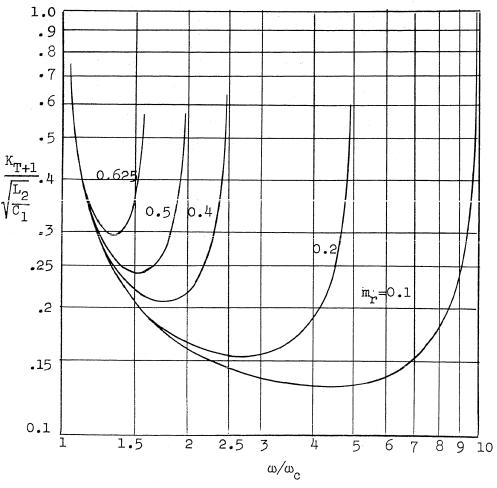


FIG. 6.10.--Transverse interaction impedance  $K_{T+1}$  of the three-element capacitively-coupled band-pass circuit.

- 62 -

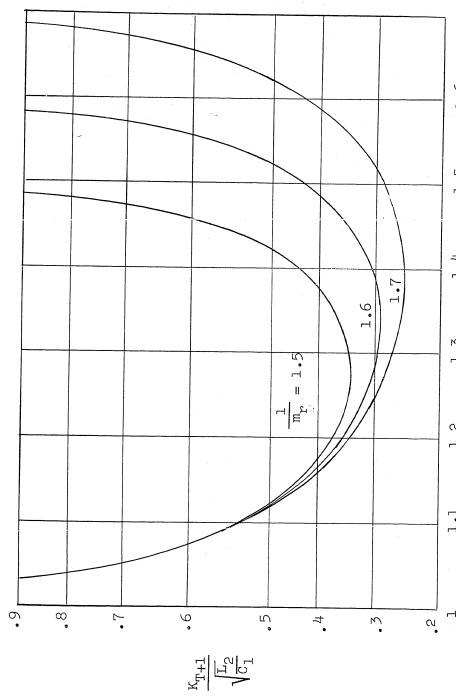


FIG. 6.11.--Transverse interaction impedance  $K_{T+1}$  of the three element capacitively-coupled band-pass circuit (enlarged scale).

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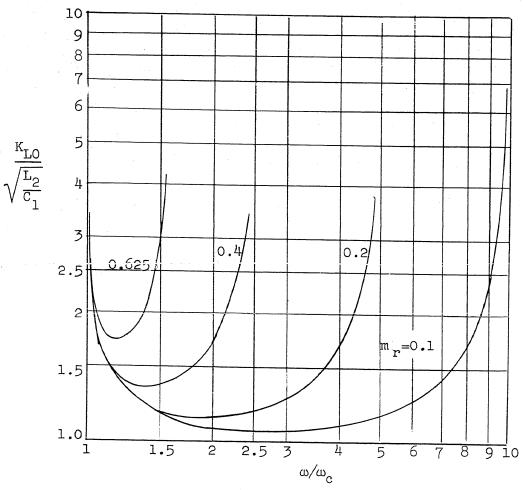


FIG. 6.12.--Longitudinal interaction impedance  $K_{L0}$  of the three-element capacitively-coupled band-pass circuit.

- 64 -

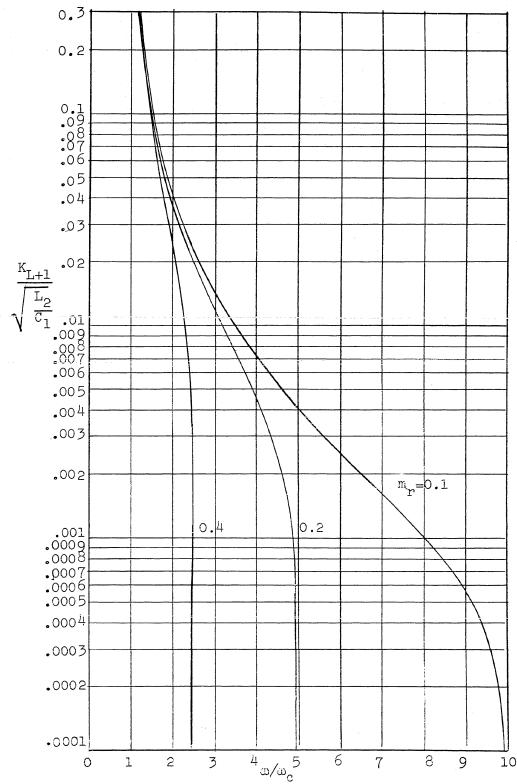


FIG. 6.13.--Longitudinal interaction impedance  $K_{L+1}$  of the three-element capacitively-coupled band-pass circuit.

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VII. CHARACTERISTICS OF THE FOUR-ELEMENT BAND-PASS CIRCUIT

The four-element band-pass circuits are shown in Fig. 7.1. The equations for the characteristics are

$$\theta = \pm 2 \sin^{-1} \sqrt{\frac{x^2 - x_1^2}{(1-x_1^2) + r(1-x^2)}} \quad (7.1)$$

$$y_{pn} = \frac{2x}{|1+2\pi f|} \quad (7.2)$$

$$y_g = \frac{\sqrt{1-x^2} \sqrt{x^2 - x_1^2}}{x \sqrt{1+r}} \quad (7.3)$$

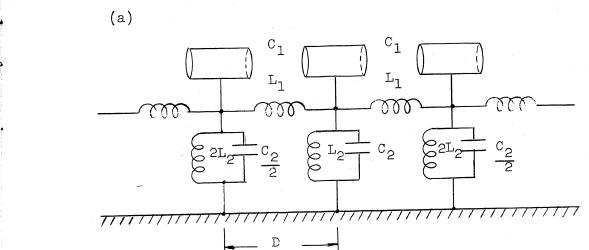
$$z_{Op} = \sqrt{\frac{L_2}{C_1}} \frac{\sqrt{r} x}{\sqrt{x^2 - x_1^2} \sqrt{1+s} \sqrt{1-x^2}} \quad (7.4)$$

$$z_{Op} = \sqrt{\frac{L_2}{C_1}} \sqrt{\frac{r}{s}} \frac{x}{x_1} \sqrt{\frac{(s+1)-(r+1)}{s} \frac{x_1^2}{x^2}} \quad (7.5)$$

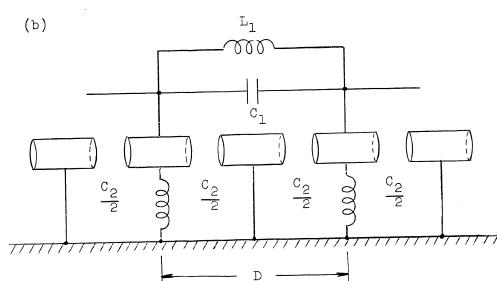
$$K_{Ln} = z_{Op} \left( \frac{\sin \frac{\beta_n D}{2}}{\frac{\beta_n D}{2}} \right)^2 \quad (7.6)$$

$$K_{Tn} = z_{Op} \frac{1}{4} \left( \frac{\sin \frac{\beta_n D}{4}}{\frac{\beta_n D}{4}} \right)^2 \quad (7.7)$$

$$\omega_c = \frac{1}{\sqrt{L_2 C_2}} \quad \frac{\sqrt{1+s}}{\sqrt{1+r}} \quad , \quad x_1 = \frac{m_s}{m_r} \quad (7.8)$$



Longitudinal interaction structure.



Transverse interaction structure.

FIG. 7.1---Four-element band-pass circuits.

where

$$m_r = \frac{1}{\sqrt{1+r}} = \frac{1}{\sqrt{1 + \frac{4C_1}{C_2}}}$$

$$m_s = \frac{1}{\sqrt{1+s}} = \frac{1}{\sqrt{1 + \frac{4L_2}{L_1}}}$$

For the fundamental-backward-wave circuit,  $x_1$  is greater than unity and the negative sign in the  $\theta$  expression should be used. For the fundamental-forward-wave circuit,  $x_1$  is less than unity and the positive sign in the  $\theta$  expression should be used.

For the case of negative mutual coupling,  $L_1$  is negative and  $L_2$  is positive. The circuit is then fundamentally a backward-wave circuit. When  $L_1$  and  $L_2$  are both positive, i.e., for the case of positive mutual coupling, the circuit can fundamentally be a forward-wave or a backward-wave circuit depending on whether  $x_1$  is less than unity or greater than unity.

Curves are presented in Figs. 7.2 to 7.91 for both the forward-wave and the backward-wave circuits.

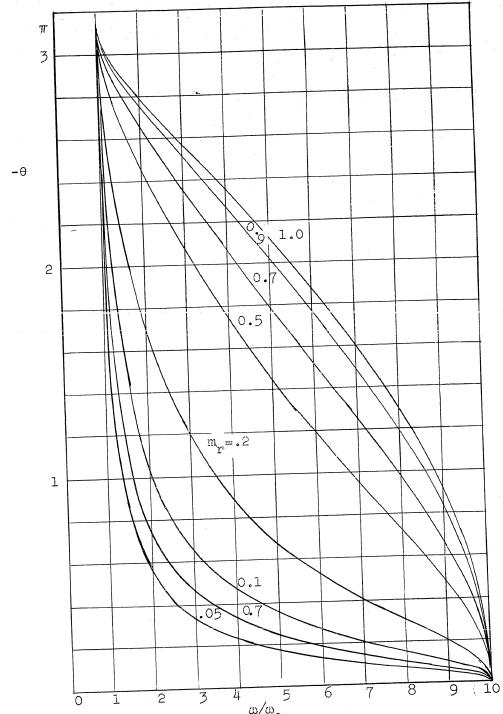


FIG. 7.2.--Phase angle per section of the four-element band-pass circuit,  $x_1=5$ .

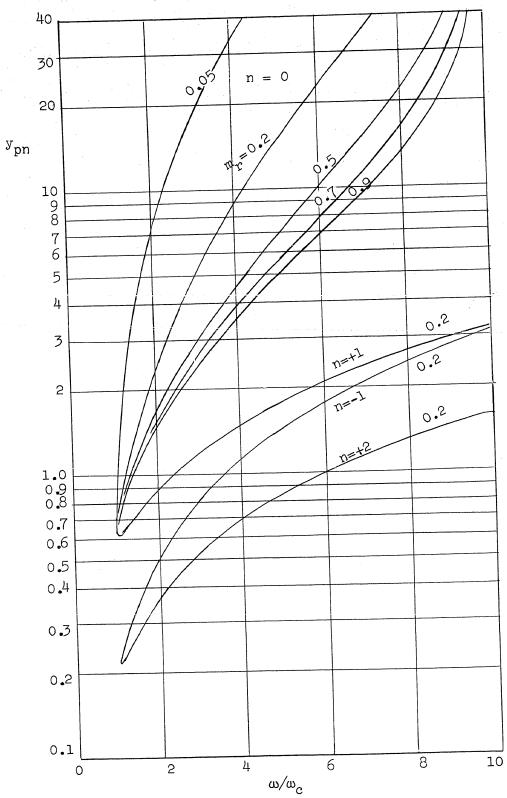


FIG. 7.3.--Phase velocity  $y_{pn}$  of the four-element band-pass circuit,  $x_1=10$ .

- 70 -

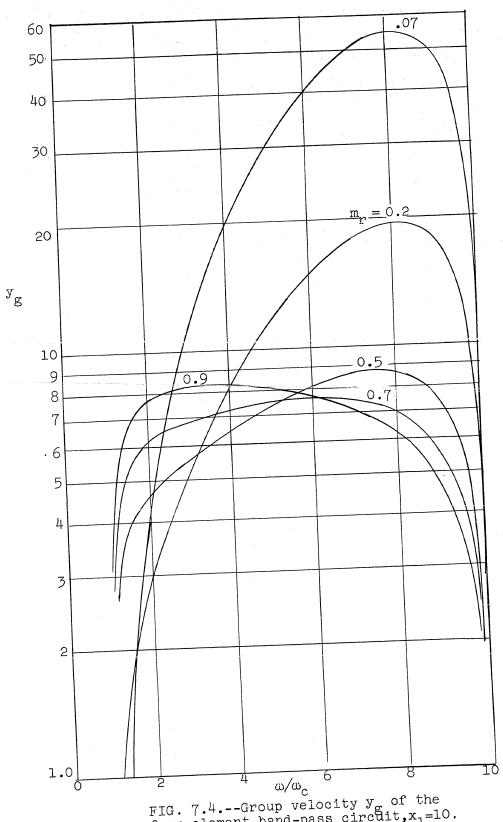


FIG. 7.4.--Group velocity  $y_g$  of the four-element band-pass circuit,  $x_1=10$ .

- 71 -

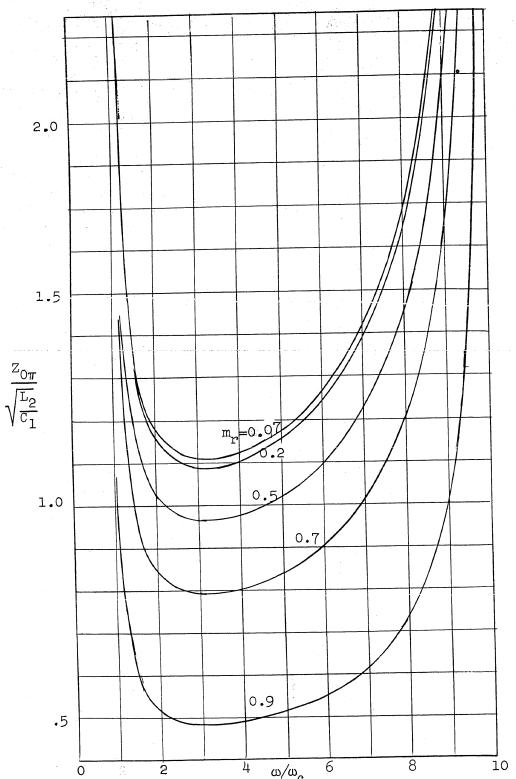


FIG. 7.5.--Circuit impedance  $Z_{0T}$  of the four-element band-pass circuit of Pi section,  $x_1 = 10$ .

- 72 -

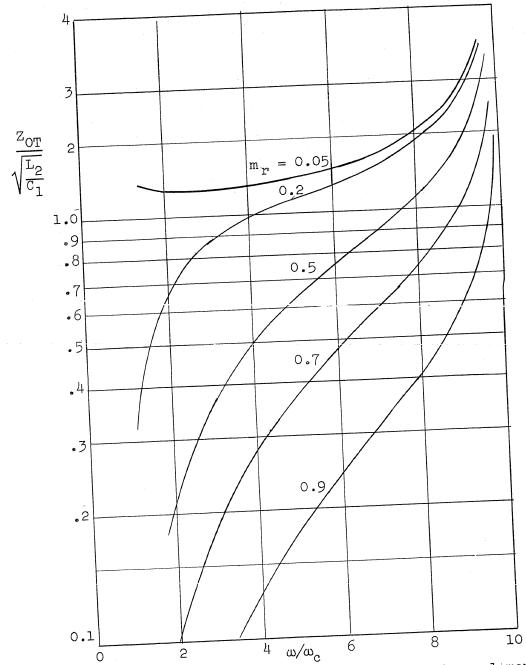


FIG. 7.6.--Circuit impedance  $Z_{0T}$  of the four-element band-pass circuit of Tee section,  $x_1 = 10$ .

- 73 -

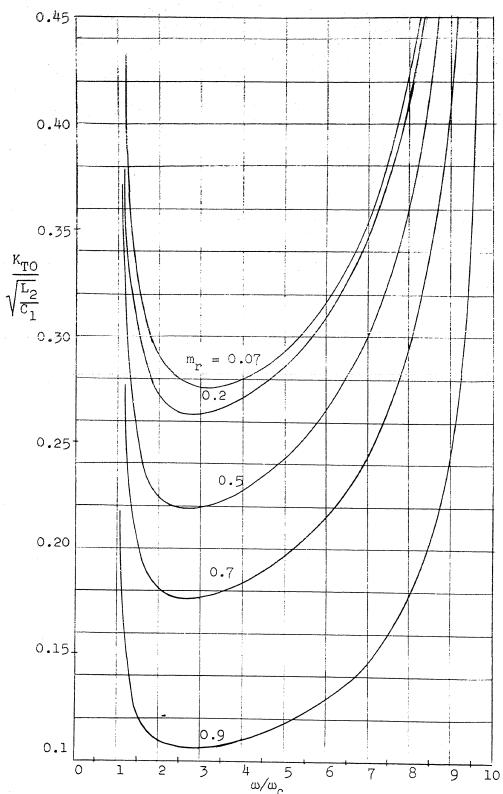


FIG. 7.7.--Transverse interaction impedance  $K_{T0}$  of the four-element band-pass circuit,  $x_1 = 10$ .

- 74 -

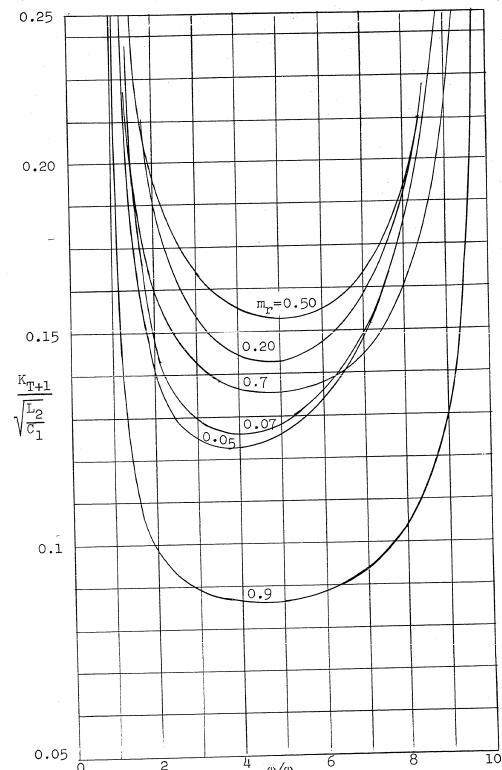


FIG. 7.8.--Transverse interaction impedance  $K_{T+1}$  of the four-element band-pass circuit,  $x_1 = 10$ .

- 75 -

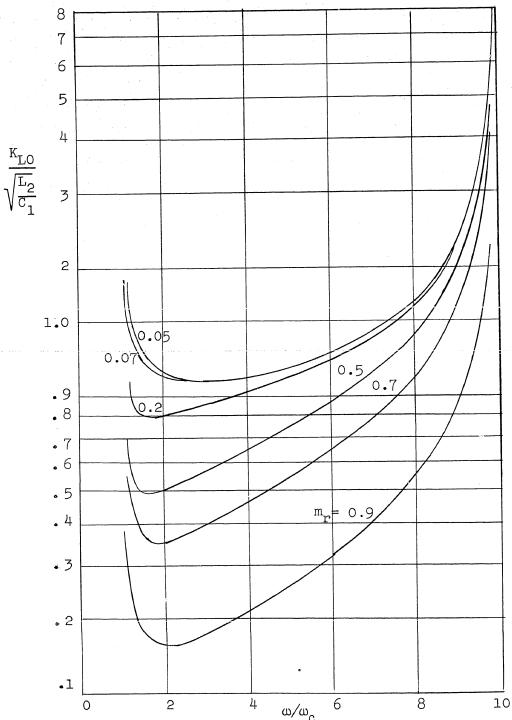


FIG. 7.9.--Longitudinal interaction impedance  $K_{L0}$  of the four-element band-pass circuit,  $x_1 = 10$ .

- 76 -

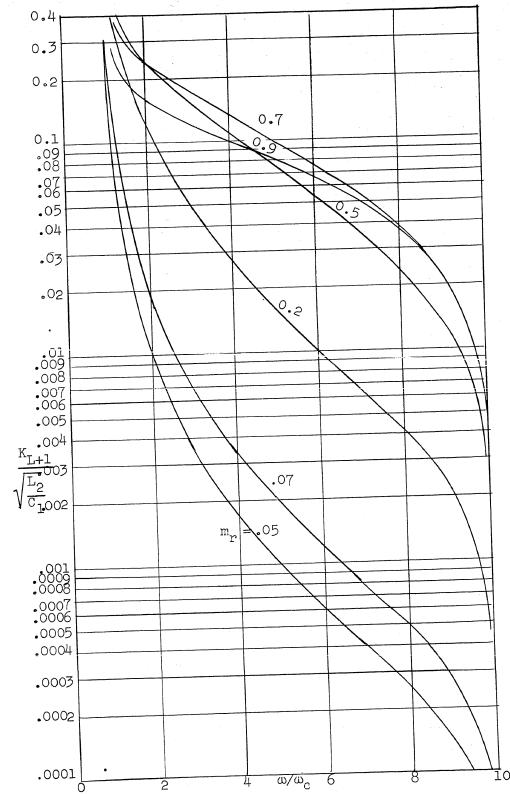


FIG. 7.10.--Longitudinal interaction impedance  $K_{L+1}$  of the four-element band-pass circuit,  $x_1 = 10$ .

- 77 -

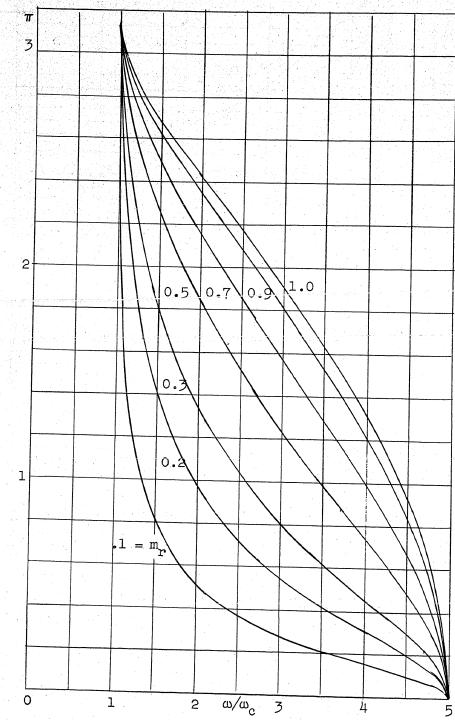


FIG. 7.11.--Phase angle per section of the four-element band-pass circuit,  $x_1 = 5$ .

- 78 -

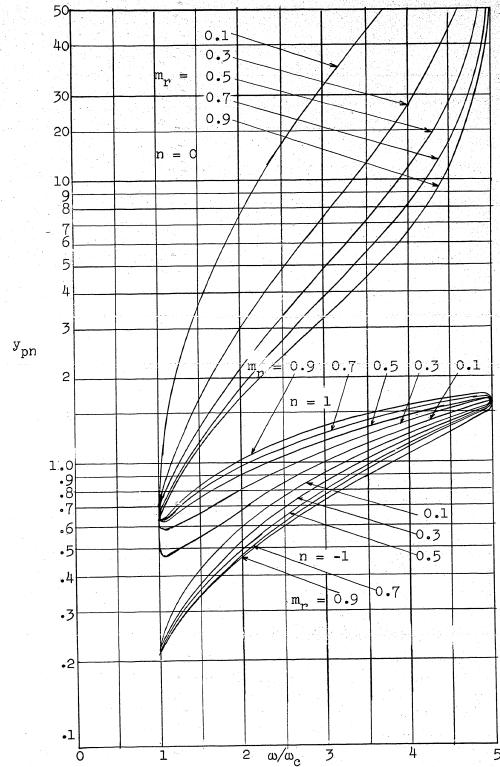


FIG. 7.12.--Phase velocity  $y_{pn}$  of the four-element band-pass circuit,  $x_1 = 5$ .

- 79 -

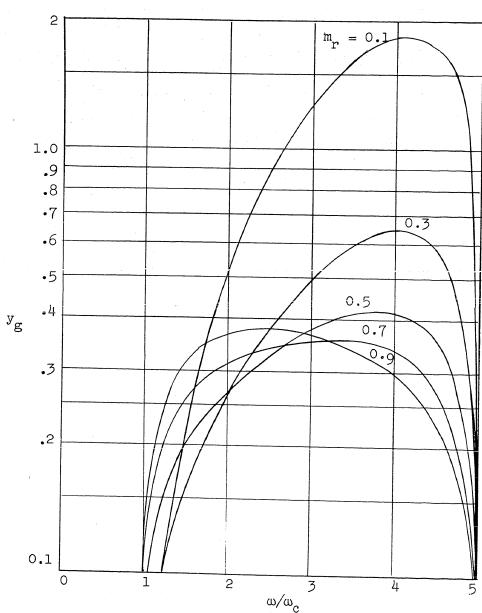


FIG. 7.13.--Group velocity  $y_g$  of the four-element band-pass circuit,  $x_1=5$ .

- 80 -

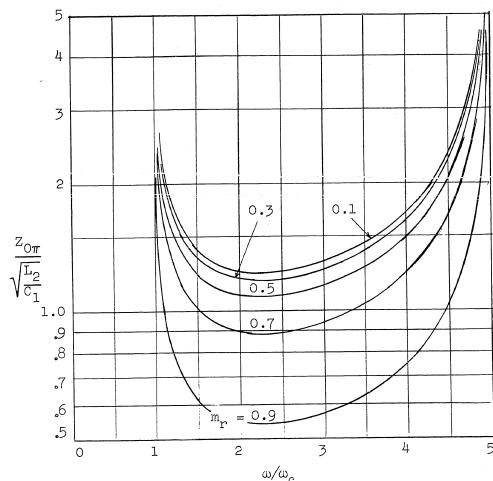


FIG. 7.14.--Circuit impedance  $Z_{Q\pi}$  of the four-element band-pass circuit of PI section,  $x_1=5$ .

- 81 -

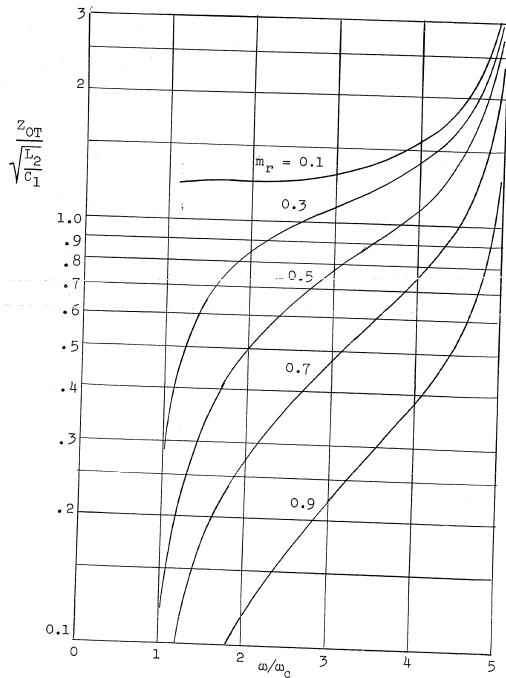


FIG. 7.15.--Circuit impedance  $Z_{0T}$  of the four-element band-pass circuit of Tee section,  $x_1 = 5$ .

- 82 -

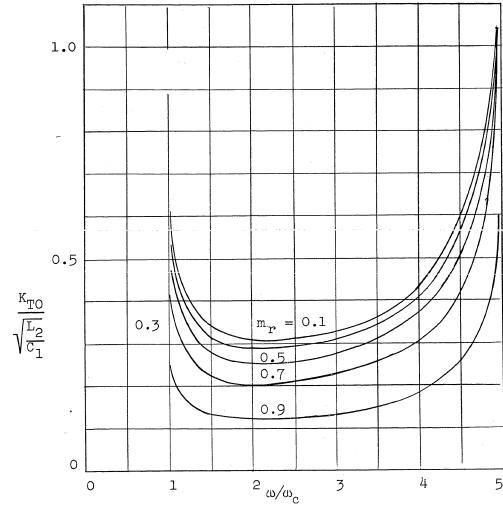


FIG. 7.16.--Transverse interaction impedance  $K_{T0}$  of the four-element band-pass circuit,  $x_1 = 5$ .

- 83 -

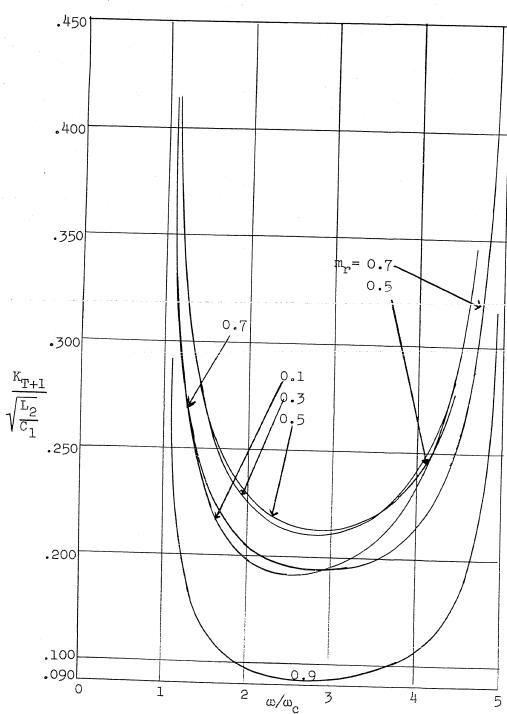


FIG. 7.17.--Transverse interaction impedance  $K_{T+1}$  of the four-element band-pass circuit,  $x_1 = 5$ .

- 84 -

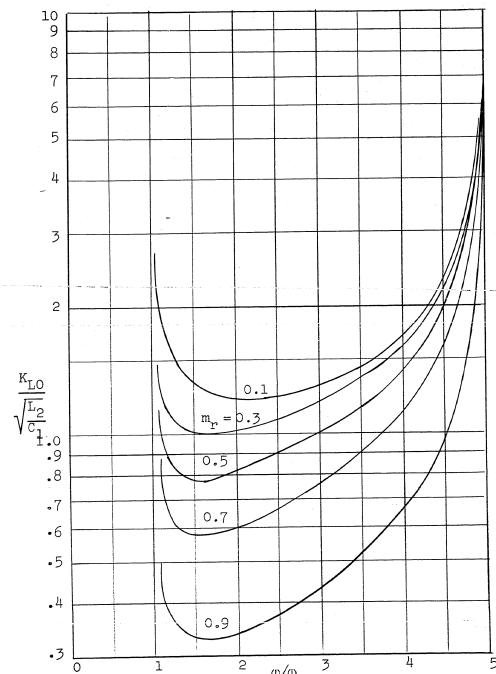
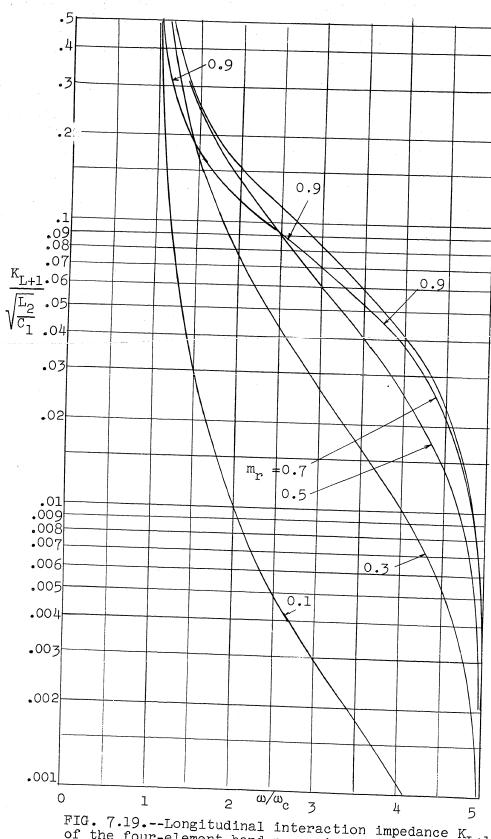
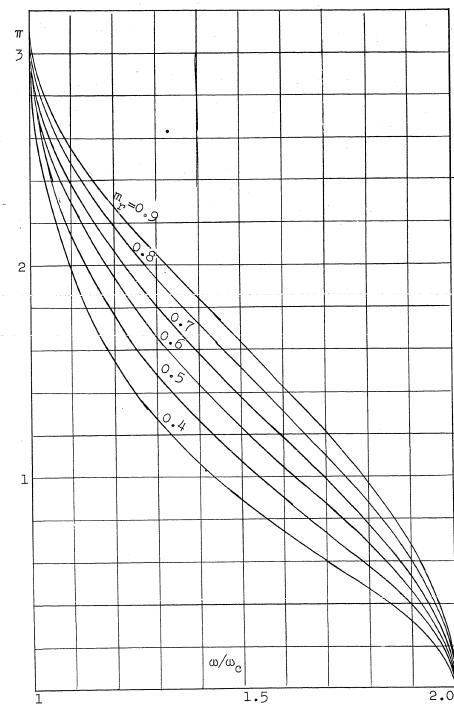


FIG. 7.18.--Longitudinal interaction impedance  $K_{L0}$  of the four-element band-pass circuit,  $x_1 = 5$ .

- 85 -



- 86 -



- 87 -

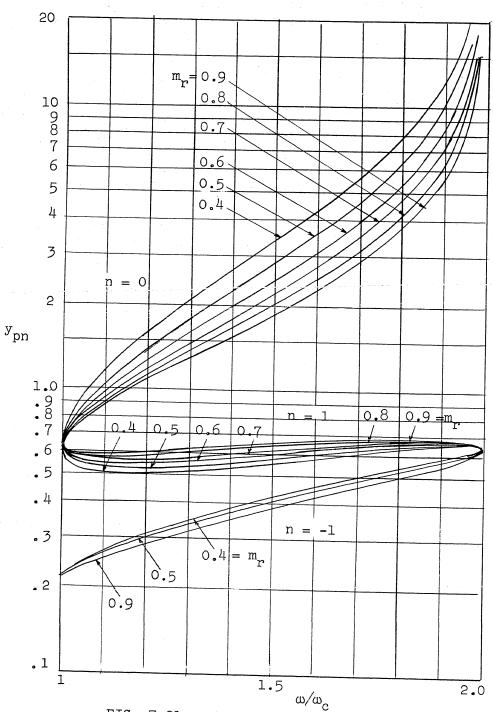


FIG. 7.21.--Phase velocity  $y_{pn}$  of the four-element band-pass circuit,  $x_1 = 2$ .

- 88 -

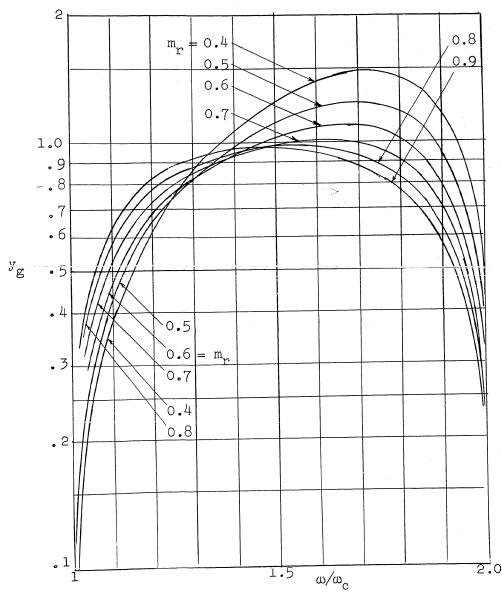


FIG. 7.22.--Group velocity  $y_g$  of the four-element band-pass circuit,  $x_1 = 2$ .

- 89 -

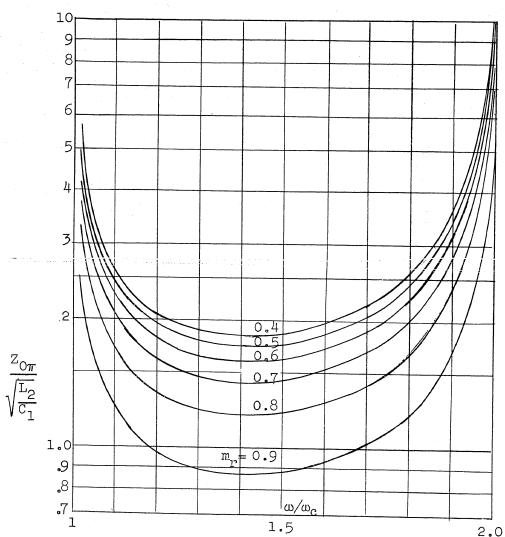


FIG. 7.23.--Circuit impedance  $Z_{0\pi}$  of the four-element band-pass circuit of Pi section,  $X_1 = 2$ .

- 90 -

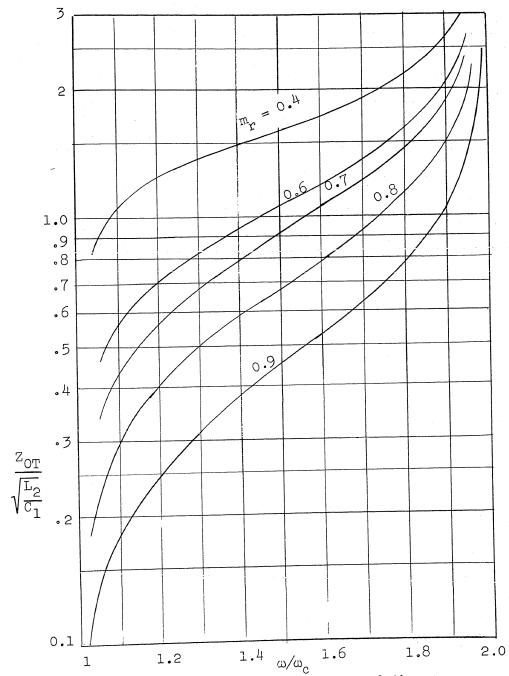


FIG. 7.24.--Circuit impedance  $Z_{0T}$  of the four-element band-pass circuit of Tee section,  $X_1 = 2$ .

- 91 -

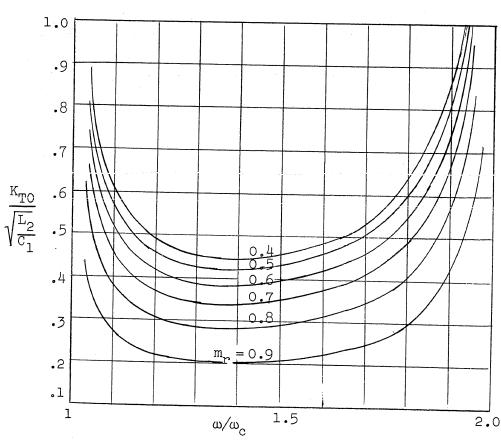


FIG. 7.25.--Transverse interaction impedance  $K_{T0}$  of the four-element band-pass circuit,  $x_1 = 2$ .

- 92 -

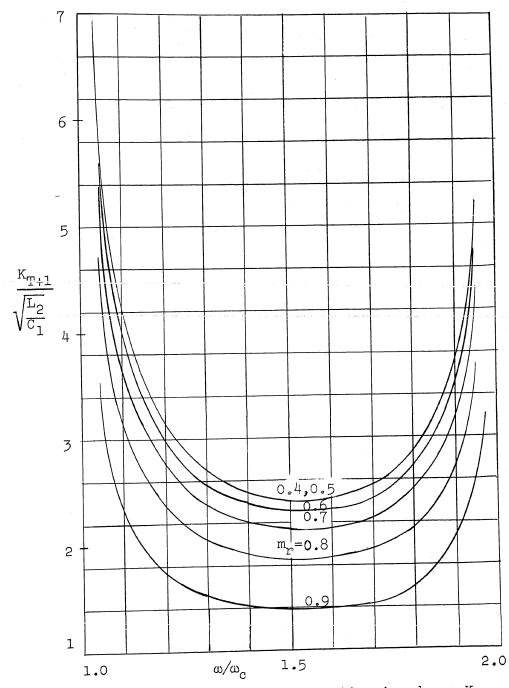


FIG. 7.26.--Transverse interaction impedance  $K_{T+1}$  of the four-element band-pass circuit,  $x_1 = 2$ .

- 93 -

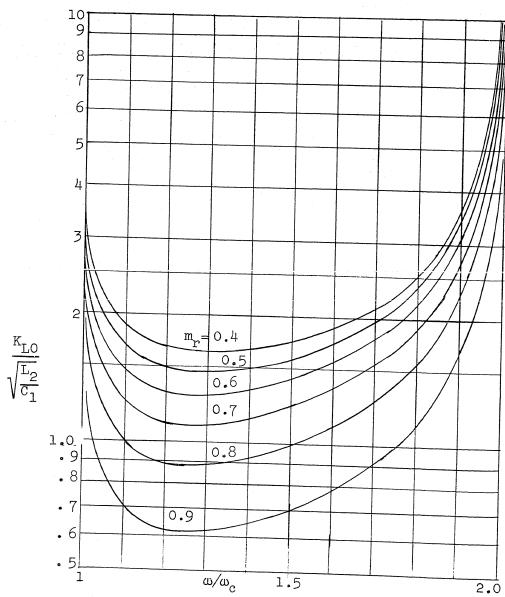


FIG. 7.27.--Longitudinal interaction impedance  $K_{L0}$  of the four-element band-pass circuit,  $x_1=2$ .

- 94 -

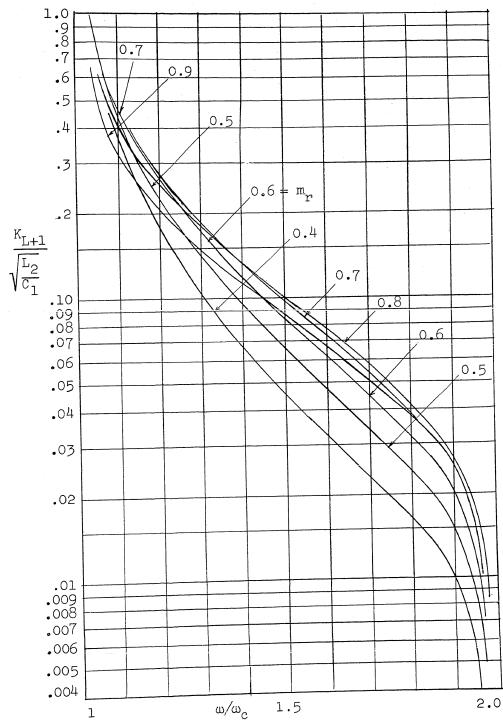


FIG. 7.28.--Longitudinal interaction impedance  $K_{L+1}$  of the four-element band-pass circuit,  $x_1=2$ .

- 95 -

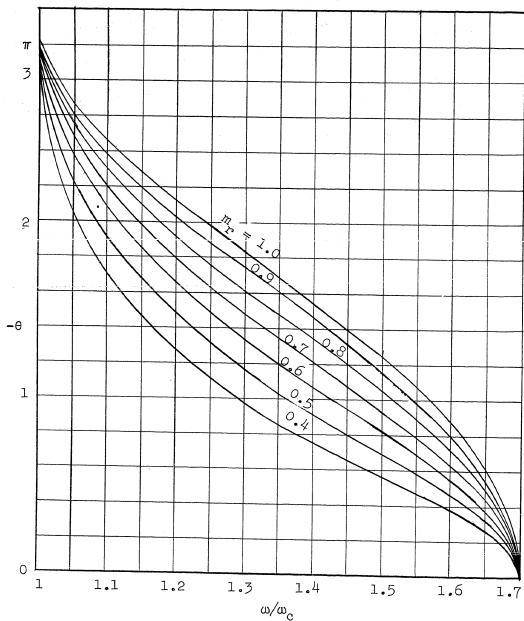


FIG. 7.29.--Phase angle per section of the four-element band-pass circuit,  $x_1 = 1.70$ .

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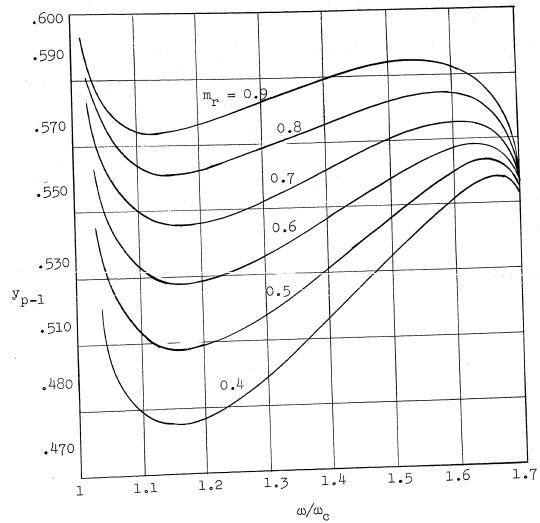


FIG. 7.30.--Phase velocity  $y_{p-1}$  of the four-element band-pass circuit,  $x_1 = 1.70$ .

- 97 -

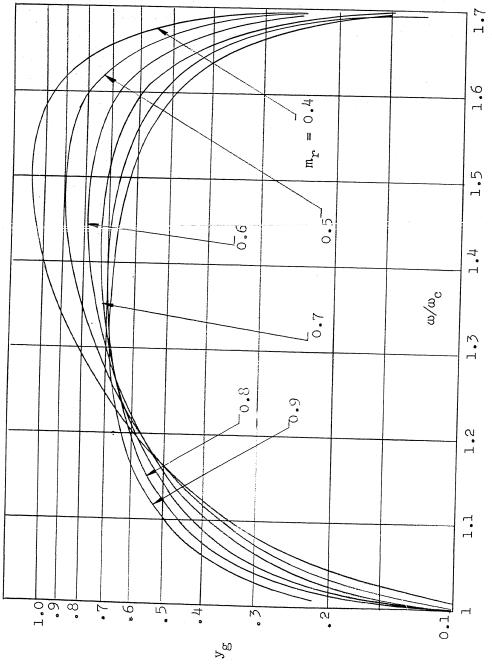


FIG. 7.31--Group velocity  $Y_g$  of the  
four-element band-pass circuit,  $x_1 = 1.70$ .

- 98 -

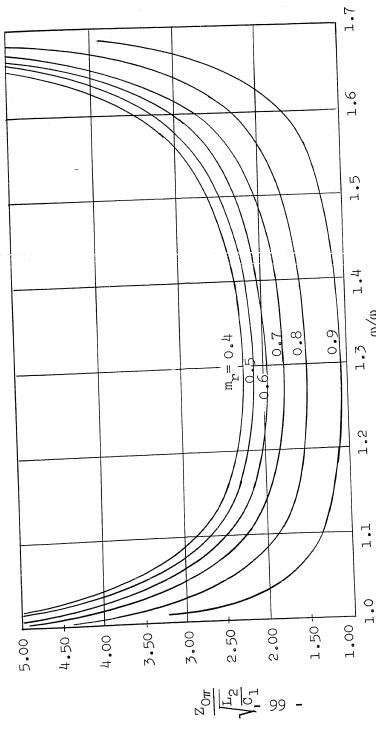


FIG. 7.32--Circuit impedance  $Z_{0r}$  of the four-element  
band-pass circuit of PI section  $x_1 = 1.70$ .

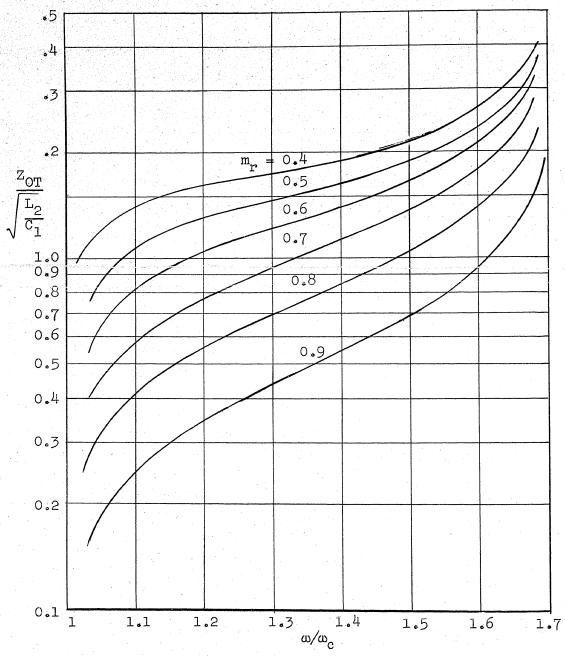


FIG. 7.33.--Circuit impedance  $Z_{OT}$  of the four-element band-pass circuit of Tee section,  $x_1=1.70$ .

- 100 -

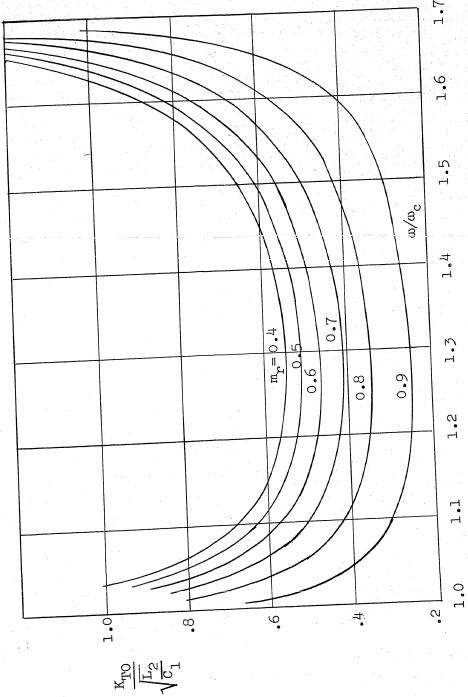


FIG. 7.34.--Transverse interaction impedance  $K_{TO}$  of the four-element band-pass circuit,  $x_1 = 1.70$ .

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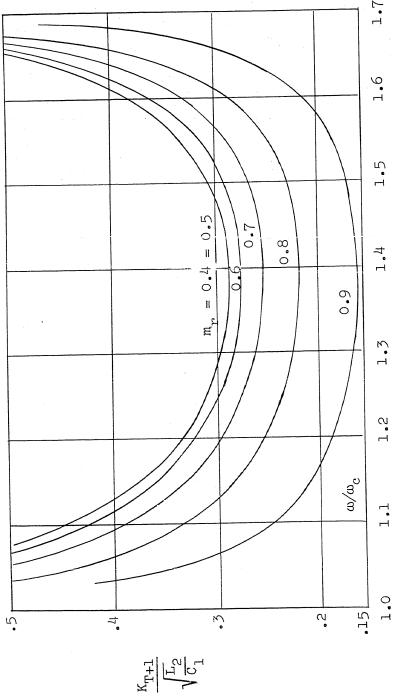


FIG. 7.25.--Transverse interaction impedance  $K_{T1+1}$   
of the four-element band-pass circuit,  $x_1 = 1.70+$

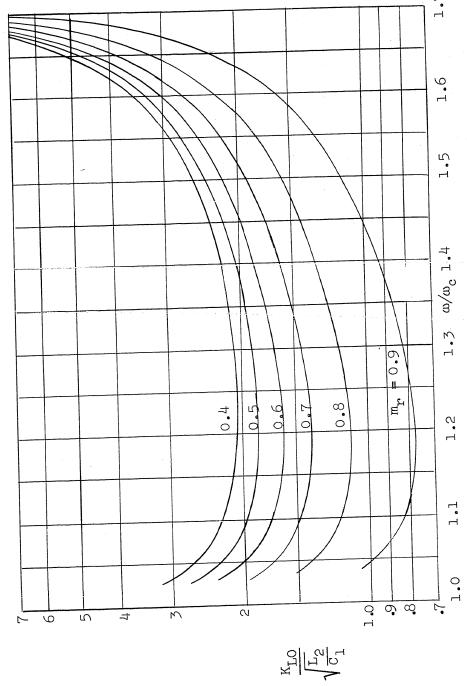
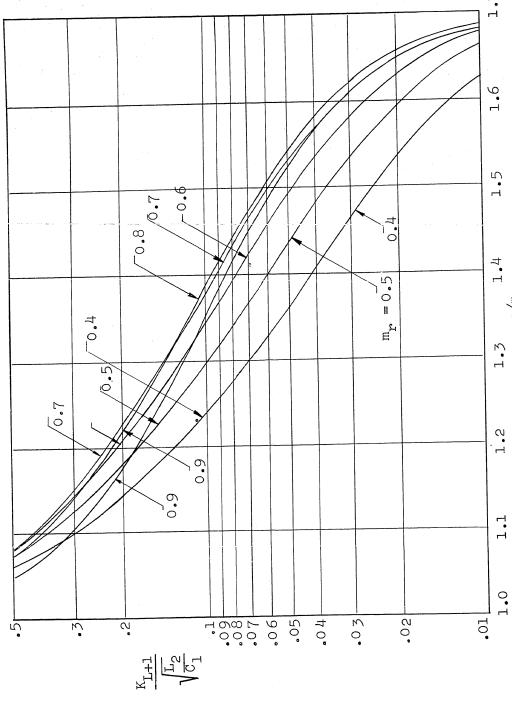


FIG. 7.26.--Longitudinal interaction impedance  
 $K_{L1}$  of the four-element band-pass circuit,  $x_1 = 1.70$ .



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FIG. 7.37.--Longitudinal interaction impedance  $K_{L+1}$  of the four-element band-pass circuit,  $x_1 = 1.70$ .

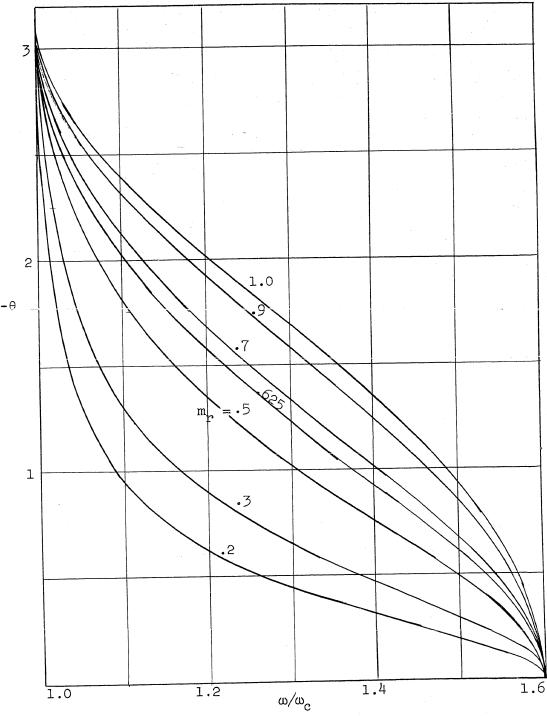


FIG. 7.38.--Phase angle per section of the four-element band-pass circuit,  $x_1 = 1.6$ .

- 105 -

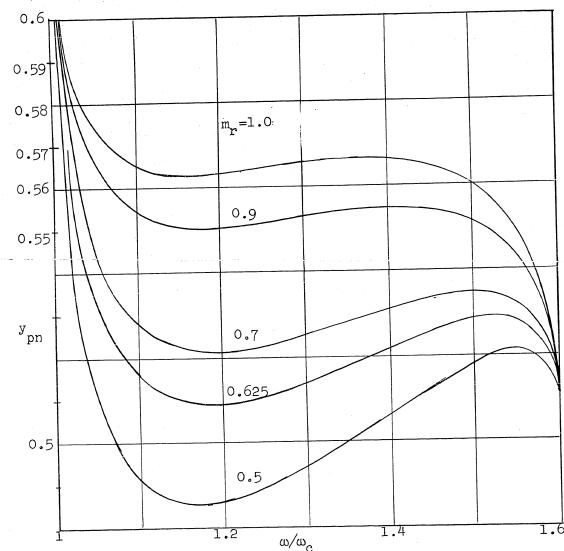


FIG. 7.39.--Phase velocity  $y_{pn}$  of the four-element band-pass circuit,  $x_1 = 1.6$ .

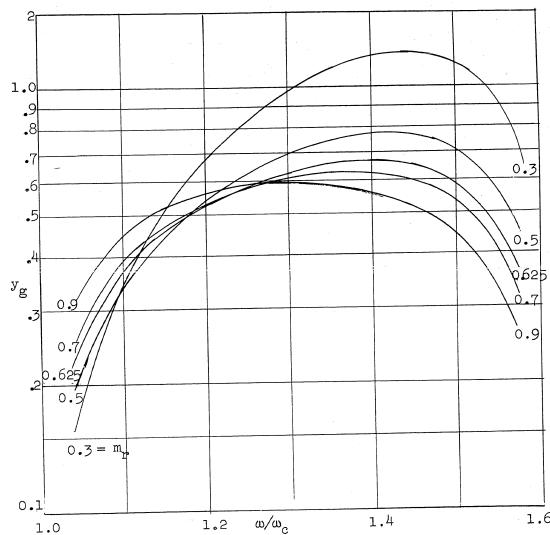


FIG. 7.40.--Group velocity  $y_g$  of the four-element band-pass circuit,  $x_1 = 1.6$ .

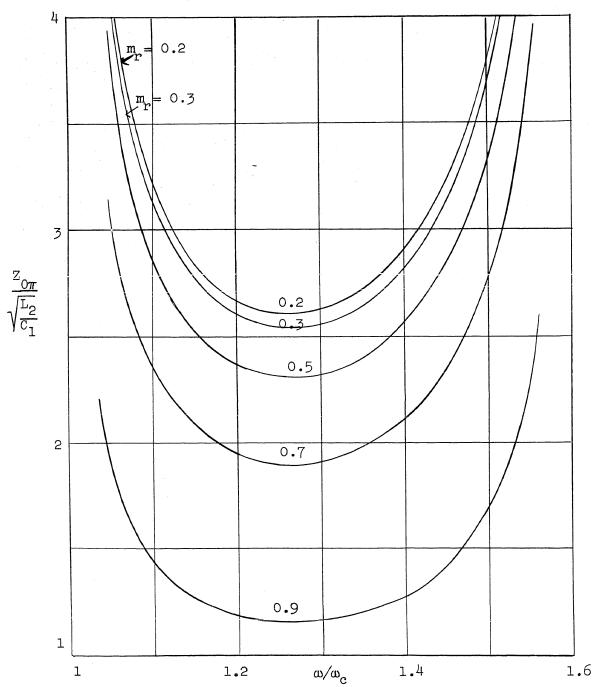


FIG. 7.41.--Circuit impedance  $Z_{OT}$  of the four-element band-pass circuit of Pi section;  $X_1 = 1.6$ .

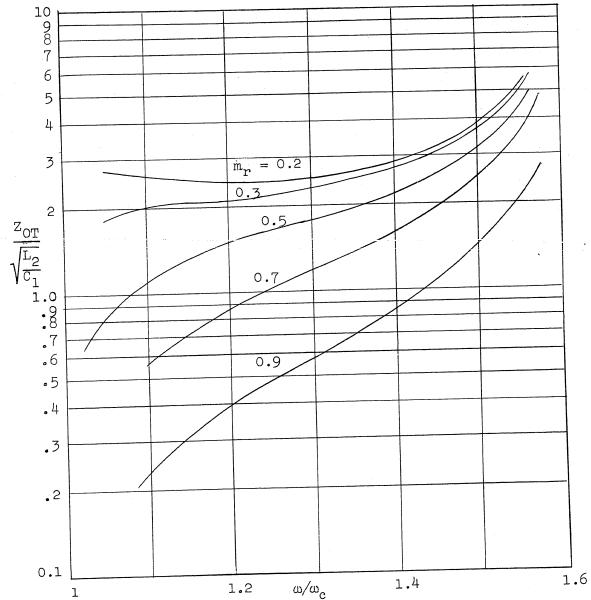


FIG. 7.42.--Circuit impedance  $Z_{OT}$  of the four-element band-pass circuit of Tee section;  $X_1 = 1.6$ .

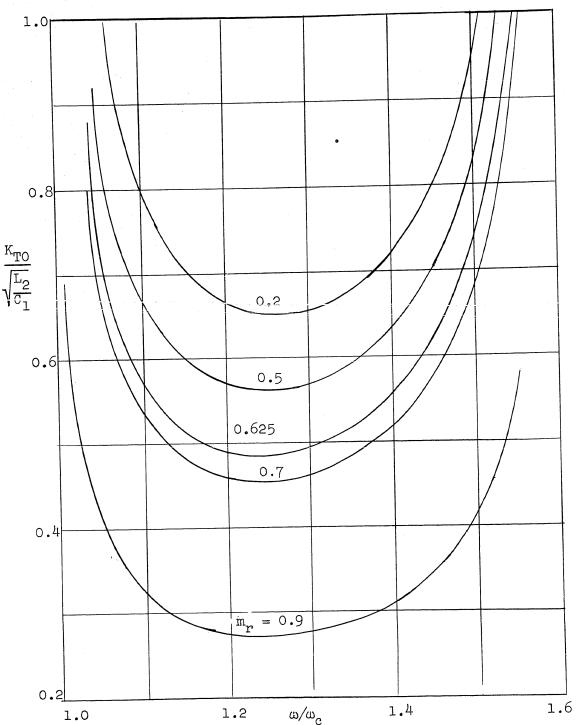


FIG. 7.43.--Transverse interaction impedance  $K_{TO}$  of the four-element band-pass circuit,  $x_1=1.6$ .

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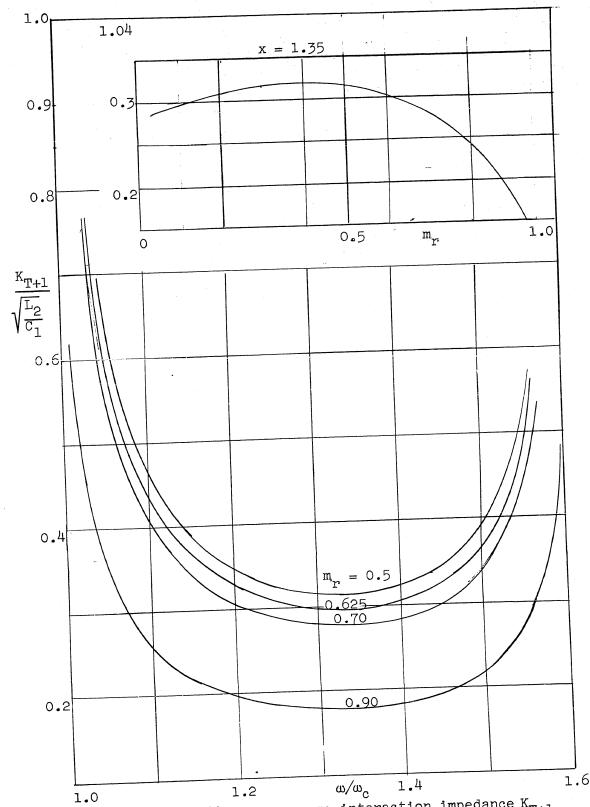


FIG. 7.44.--Transverse interaction impedance  $K_{T+1}$  of the four-element band-pass circuit,  $x_1=1.6$ .

- 111 -

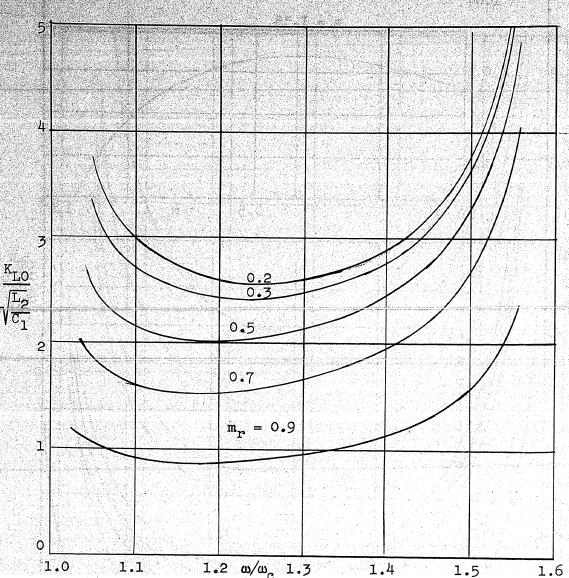


FIG. 7.45.--Longitudinal interaction impedance  $K_{L0}$  of the four-element band-pass circuit,  $x_1=1.6$ .

- 112 -

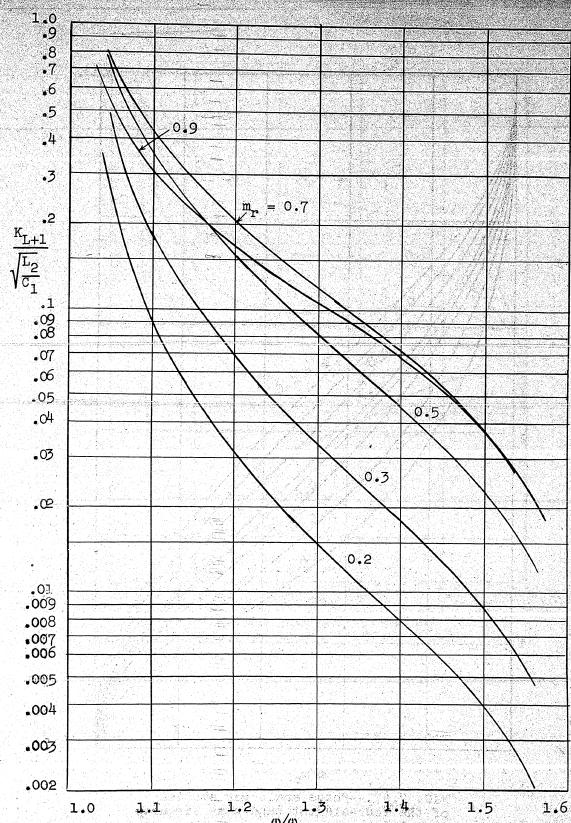


FIG. 7.46.--Longitudinal interaction impedance  $K_{L+1}$  of the four-element band-pass circuit,  $x_1 = 1.6$ .  $L+1$

- 113 -

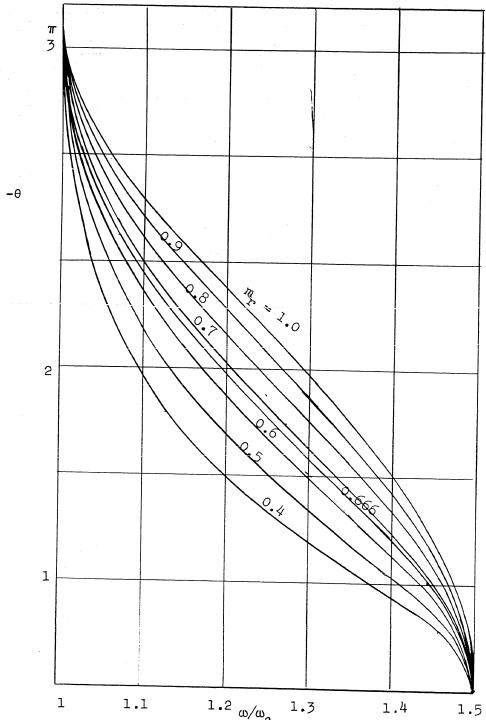


FIG. 7.47.--Phase angle per section  
of the four-element band-pass circuit,  
 $x_1 = 1.5$ .

- 114 -

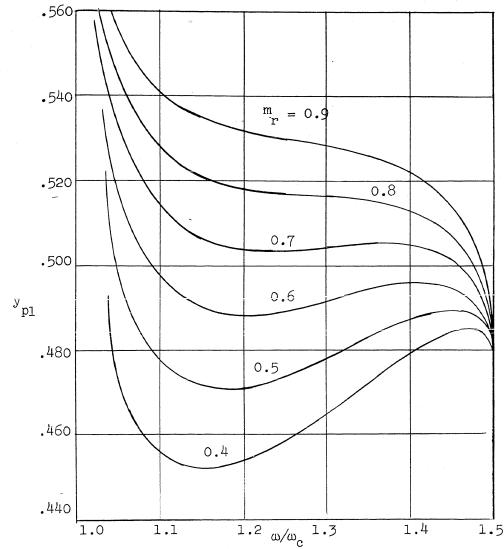


FIG. 7.48.--Phase velocity  $y_{p1}$  of the  
four-element band-pass circuit,  $x_1 = 1.5$ .

- 115 -

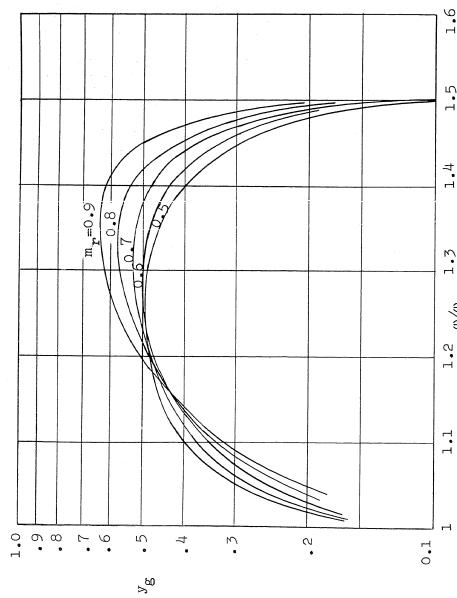


FIG. 7.49--Group velocity  $y_g$  of the four-element band-pass circuit,  $x_1 = 1.5$ .

- 116 -

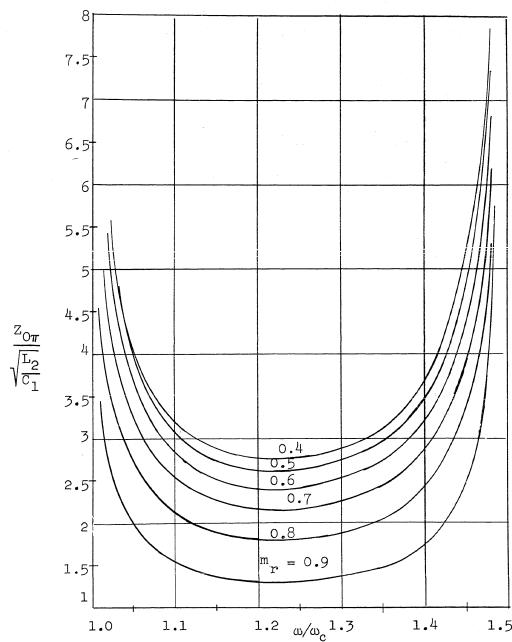


FIG. 7.50--Circuit impedance  $Z_{0\pi}$  of the four-element band-pass circuit of Pi section,  $x_1 = 1.5$ .

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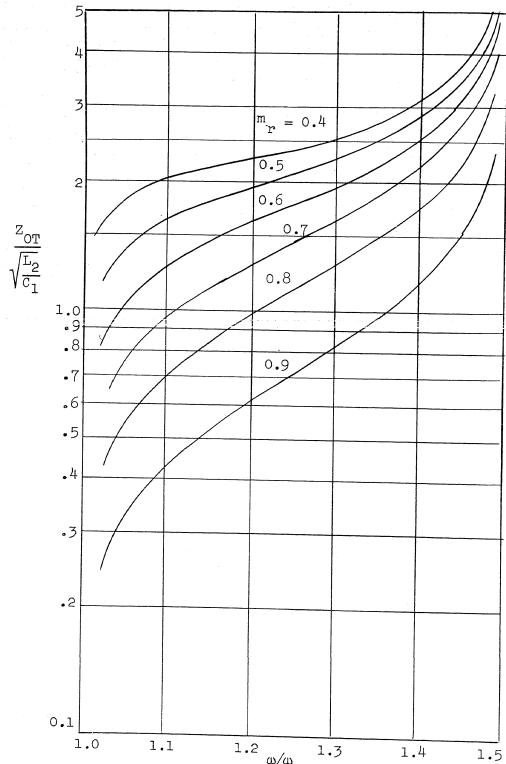


FIG. 7.51.--Circuit impedance  $Z_{OPT}$  of the four-element band-pass circuit of Tee section,  $x_1 = 1.5$ .

- 118 -

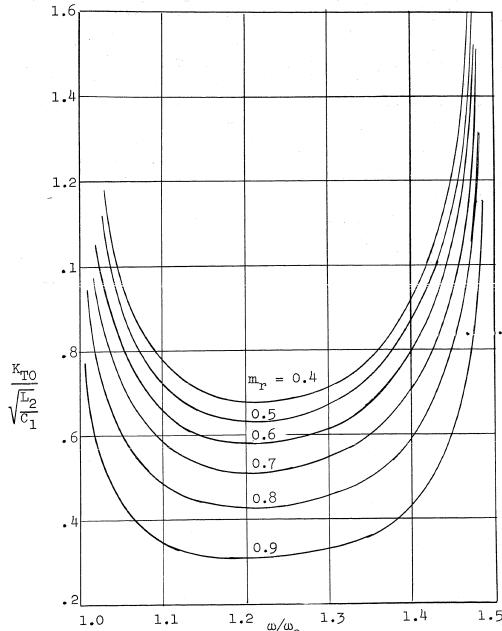


FIG. 7.52.--Transverse interaction impedance  $K_{TO}$  of the four-element band-pass circuit,  $x_1 = 1.5$ .

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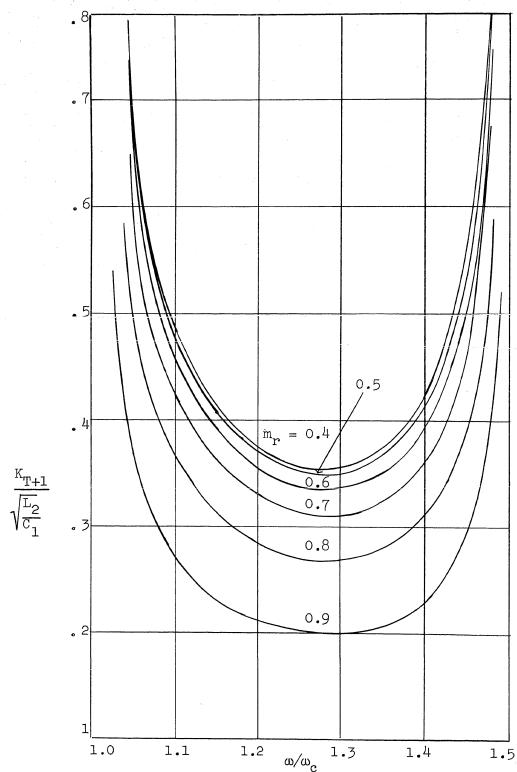


FIG. 7.53.--Transverse interaction impedance  $K_{T+1}$  of the four-element band-pass circuit,  $x_1 = 1.5$ .

- 120 -

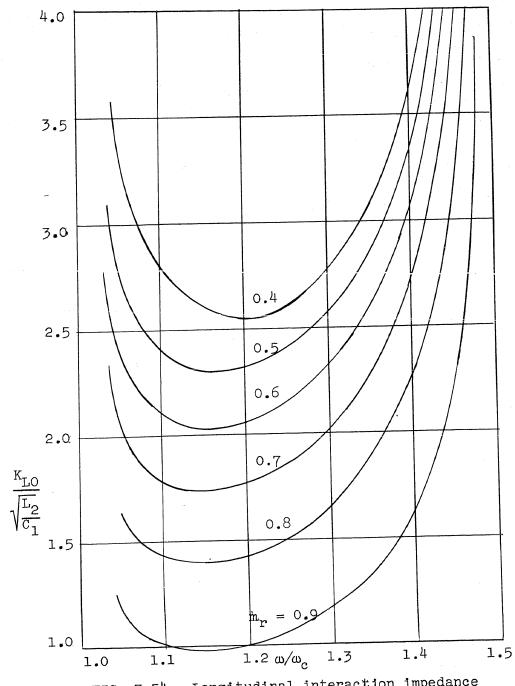


FIG. 7.54.--Longitudinal interaction impedance  $K_{LO}$  of the four-element band-pass circuit,  $x_1 = 1.5$ .

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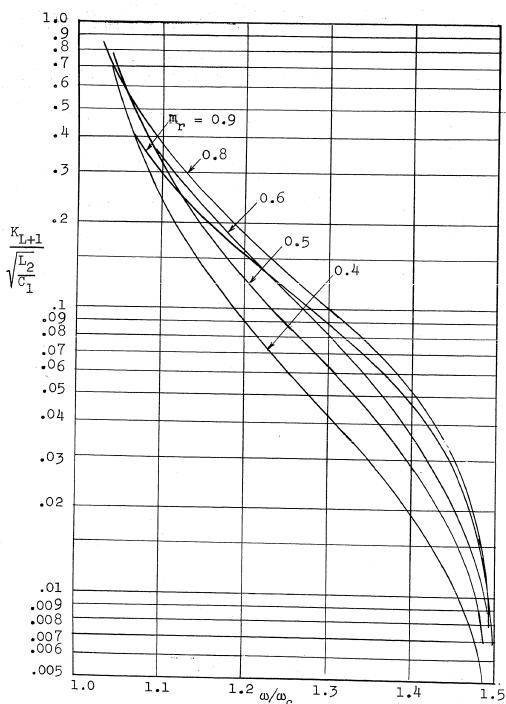


FIG. 7.55.--Longitudinal interaction impedance  $K_{L+1}$  of the four-element band-pass circuit,  $x_1 = 1.5$ .

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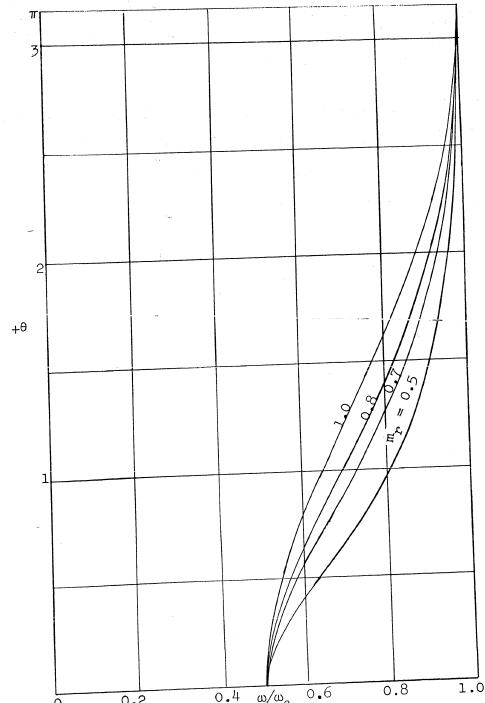


FIG. 7.56.--Phase angle per section of the four-element band-pass circuit,  $x_1 = 0.5$ .

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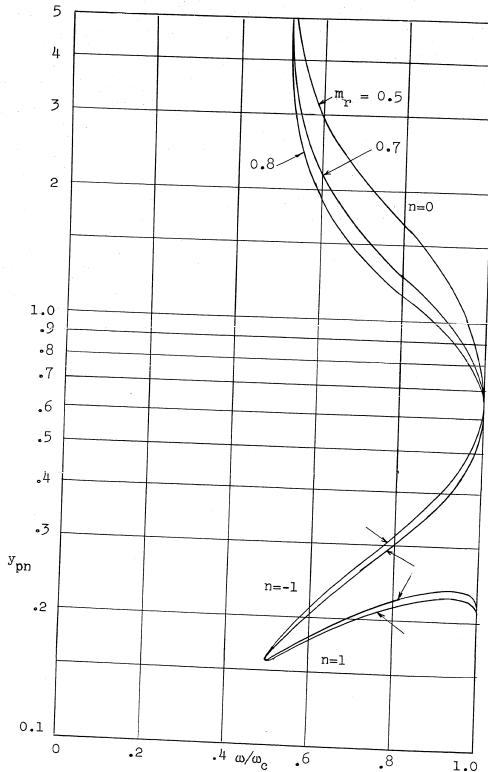


FIG. 7.57.--Phase velocity  $y_{pn}$  of the four-element band-pass circuit  $x_1 = 0.5$ .

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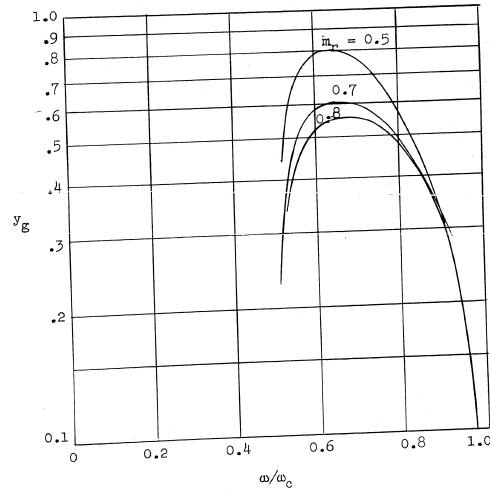


FIG. 7.58.--Group velocity  $y_g$  of the four-element band-pass circuit  $x_1 = 0.5$ .

- 125 -

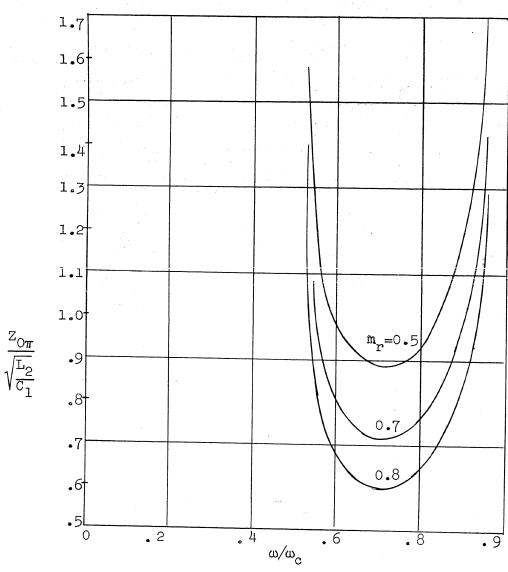


FIG. 7.59.--Circuit impedance  $Z_{0\pi}$  of the four-element band-pass circuit of Pi section,  $x_1 = 0.5$ .

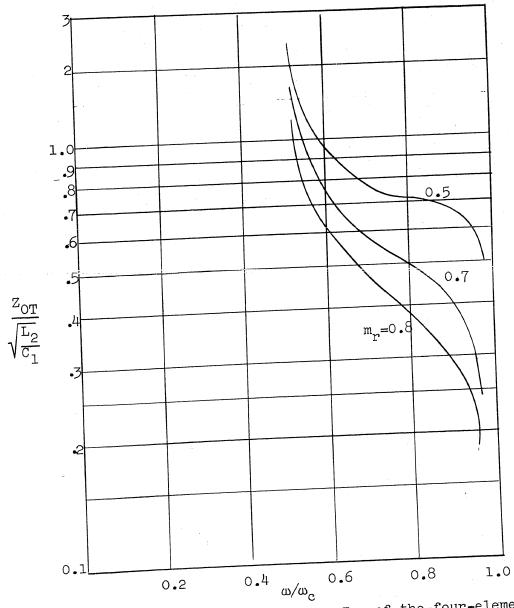


FIG. 7.60.--Circuit impedance  $Z_{0T}$  of the four-element band-pass circuit of Tee section,  $x_1 = 0.5$ .

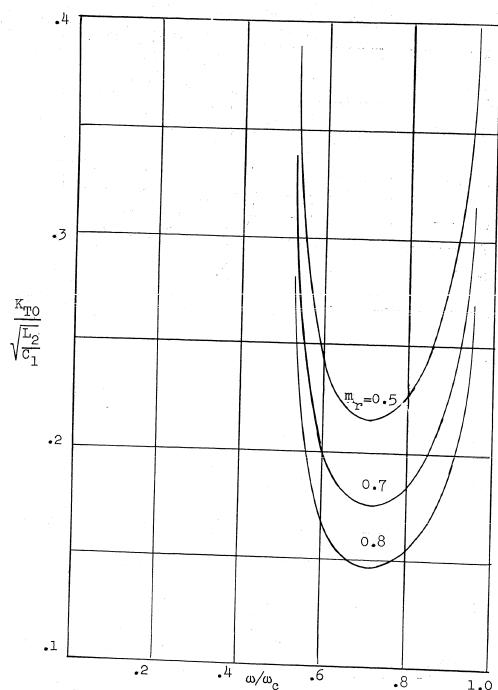


FIG. 7.61.--Transverse interaction impedance  $K_{T0}$  of the four-element band-pass circuit,  $x_1=0.5$ .

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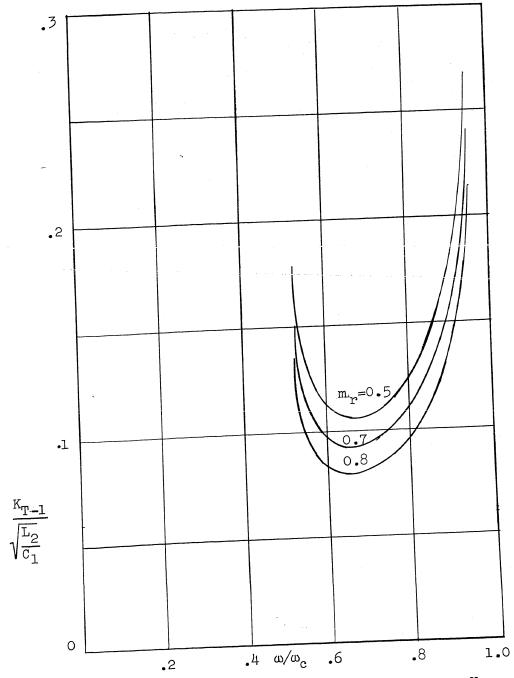


FIG. 7.62.--Transverse interaction impedance  $K_{T-1}$  of the four-element band-pass circuit,  $x_1 = 0.5$ .

- 129 -

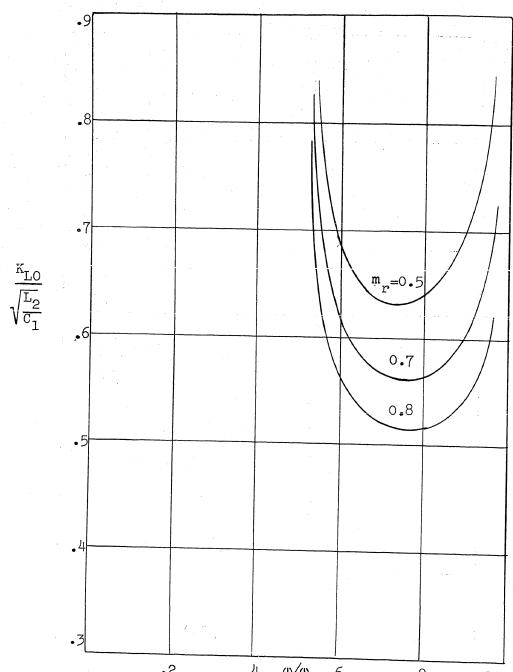


FIG. 7.63.--Longitudinal interaction impedance  $K_{L0}$  of the four-element band-pass circuit,  $x_1=0.5$ .

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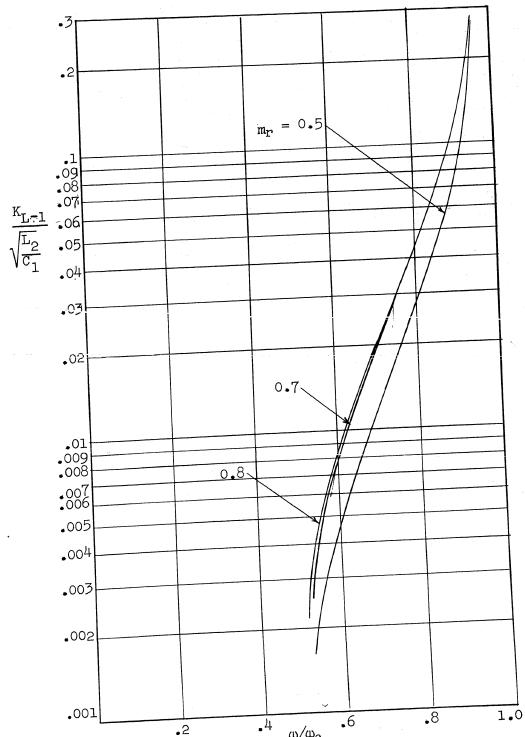


FIG. 7.64.--Longitudinal interaction impedance  $K_{L-1}$  of the four-element band-pass circuit,  $x_1=0.5$ .

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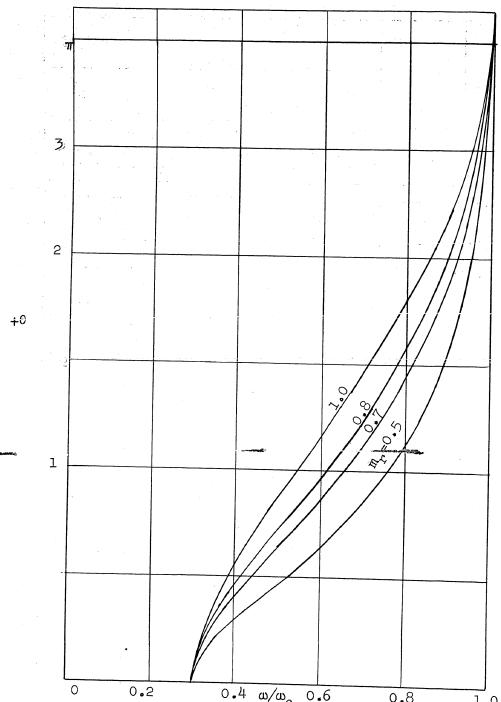


FIG. 7.65.--Phase angle per section of the four-element band-pass circuit,  $x_1 = 0.3$ .

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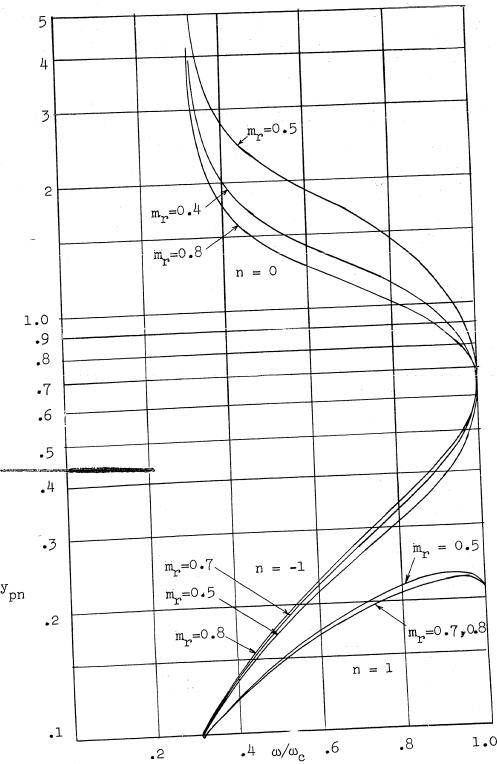


FIG. 7.66.--Phase velocity  $y_{pn}$  of the four-element band-pass circuit,  $x_1 = 0.3$ .

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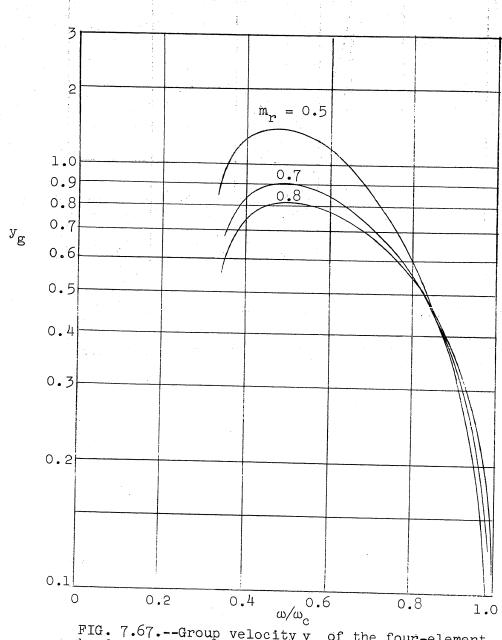


FIG. 7.67.--Group velocity  $y_g$  of the four-element band-pass circuit,  $x_1 = 0.3$ .

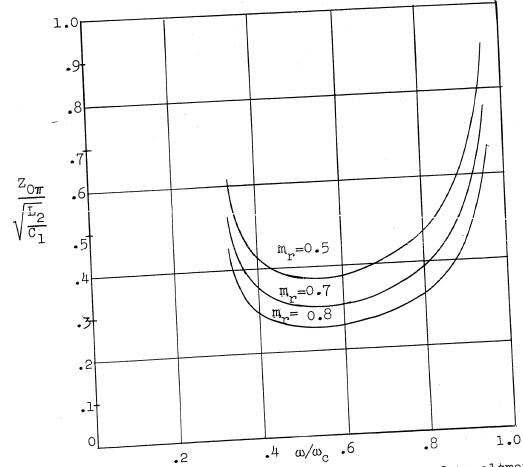


FIG. 7.68.--Circuit impedance  $Z_{0\pi}$  of the four-element band-pass circuit of Pi section,  $x_1 = 0.3$ .

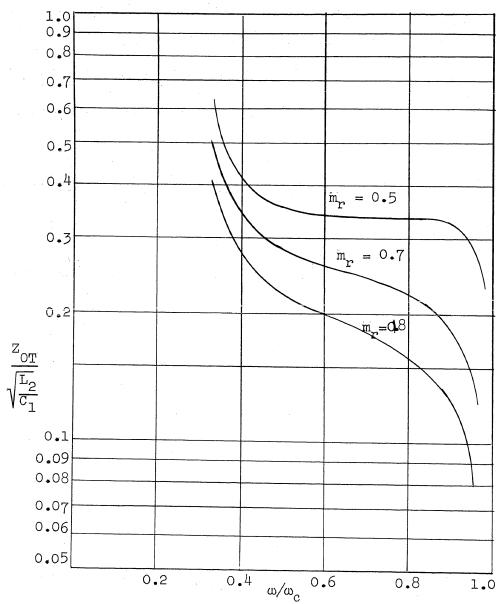


FIG. 7.69.--Circuit impedance  $Z_{OT}$  of the four-element band-pass circuit of Tee section,  $x_1 = 0.3$ .

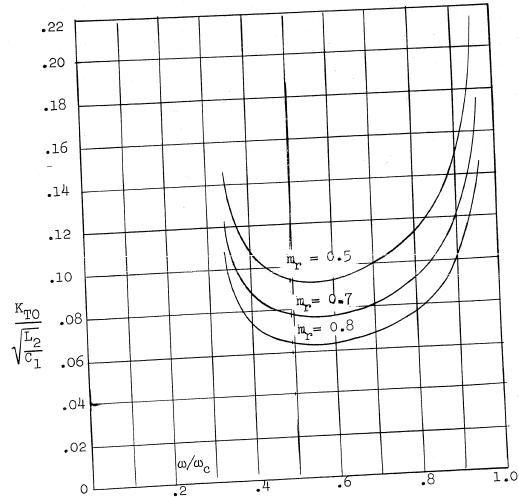


FIG. 7.70.--Transverse interaction impedance  $K_{TO}$  of the four-element band-pass circuit,  $x_1 = 0.3$ .

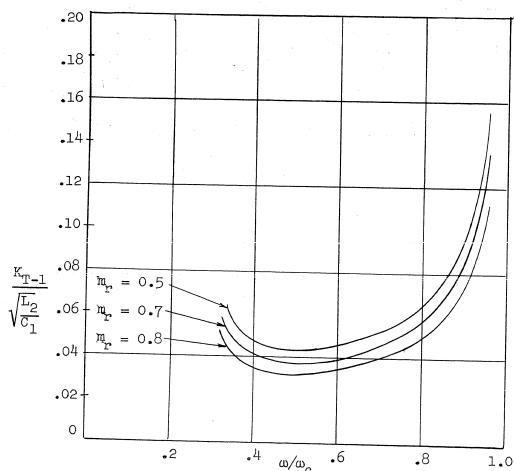


FIG. 7.71.--Transverse interaction impedance  $K_{T-1}$  of the four-element band-pass circuit,  $x_1 = 0.5$ .

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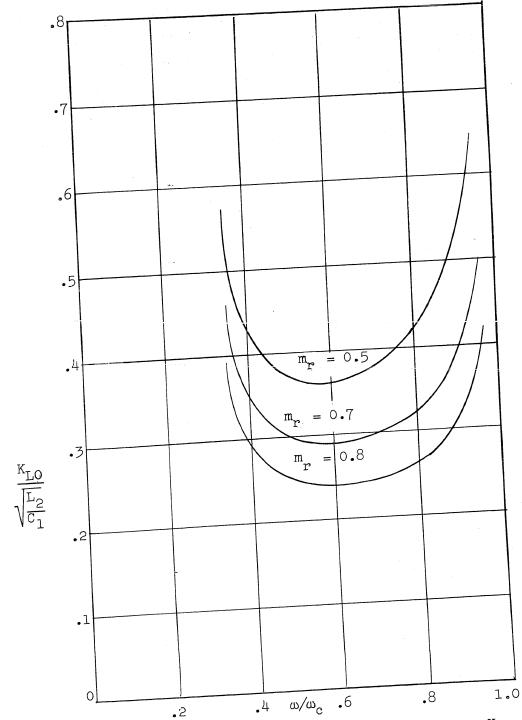


FIG. 7.72.--Longitudinal interaction impedance  $K_{LO}$  of the four-element band-pass circuit,  $x_1 = 0.3$ .

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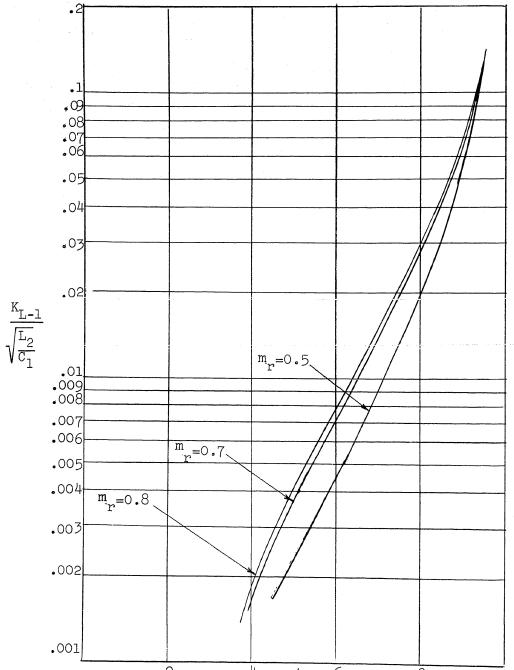


FIG. 7.73.--Longitudinal interaction impedance  $K_{L-1}$  of the four-element band-pass circuit  $x_1 = 0.3$ .

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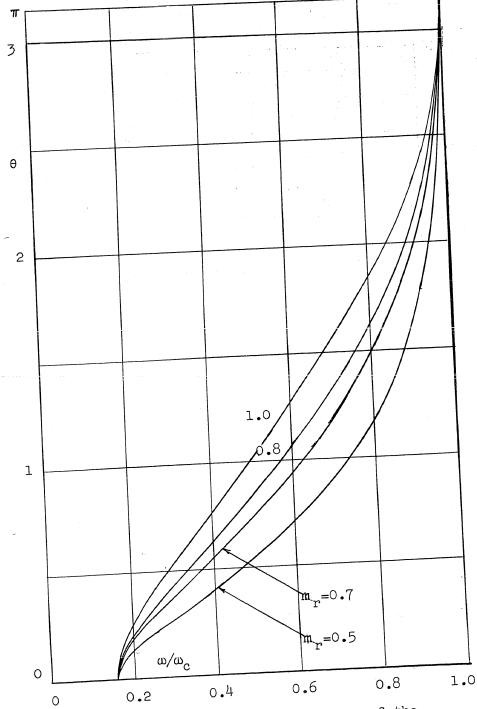
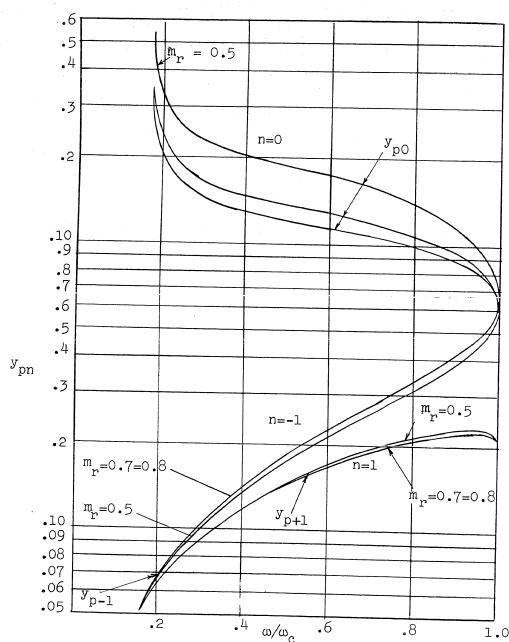
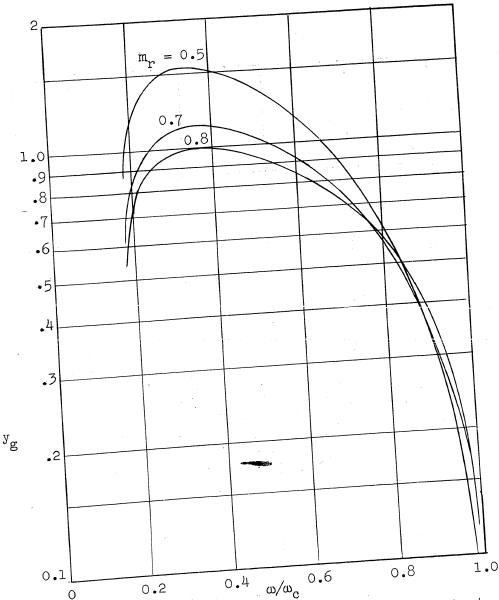


FIG. 7.74.--Phase angle per section of the four-element band-pass circuit,  $x_1 = 0.16$ .

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FIG. 7.75.--Phase velocity  $y_{pn}$  of the four-element band-pass circuit,  $x_1 = 0.16$ .FIG. 7.76.--Group velocity  $y_g$  of the four-element band-pass circuit,  $x_1 = 0.16$ .

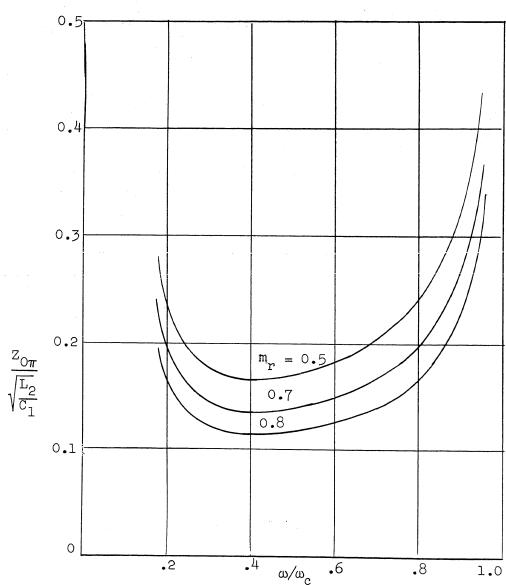


FIG. 7.77--Circuit impedance  $Z_{0\pi}$  of the four-element band-pass circuit of Pi section,  $x_1 = 0.16$ .

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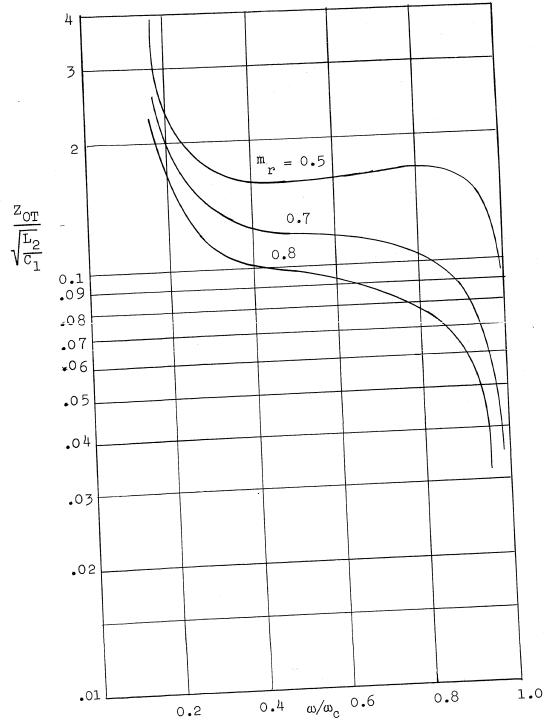


FIG. 7.78--Circuit impedance  $Z_{0\pi}$  of the four-element band-pass circuit of Tee section  $x_1 = 0.16$ .

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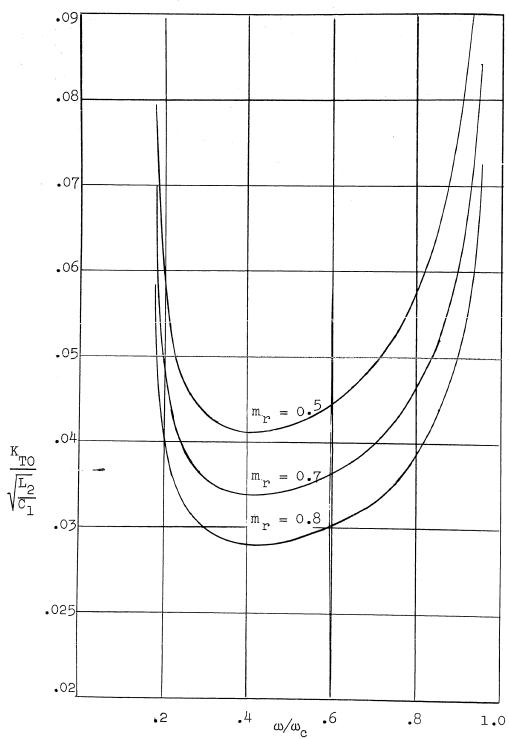


FIG. 7.79.--Transverse interaction impedance  $K_{TO}$  of the four-element band-pass circuit,  $x_1 = 0.16$ .

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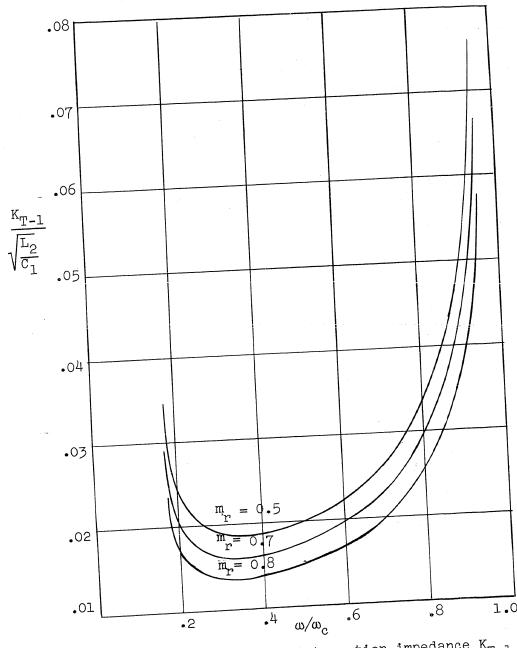


FIG. 7.80.--Transverse interaction impedance  $K_{T-1}$  of the four-element band-pass circuit,  $x_1 = 0.16$ .

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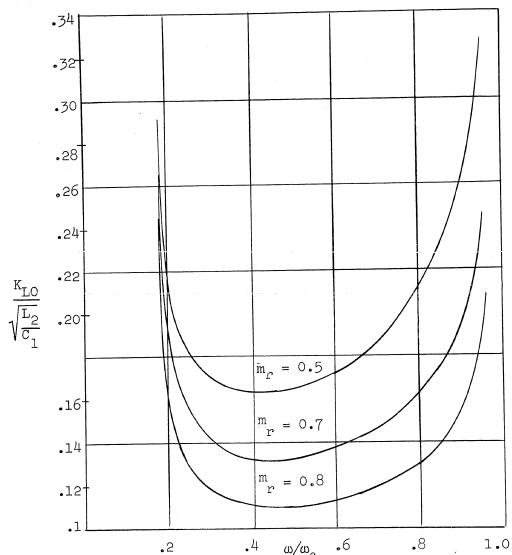


FIG. 7.81.--Longitudinal interaction impedance  $K_{L0}$  of the four-element band-pass circuit,  $x_1 = 0.16$ .

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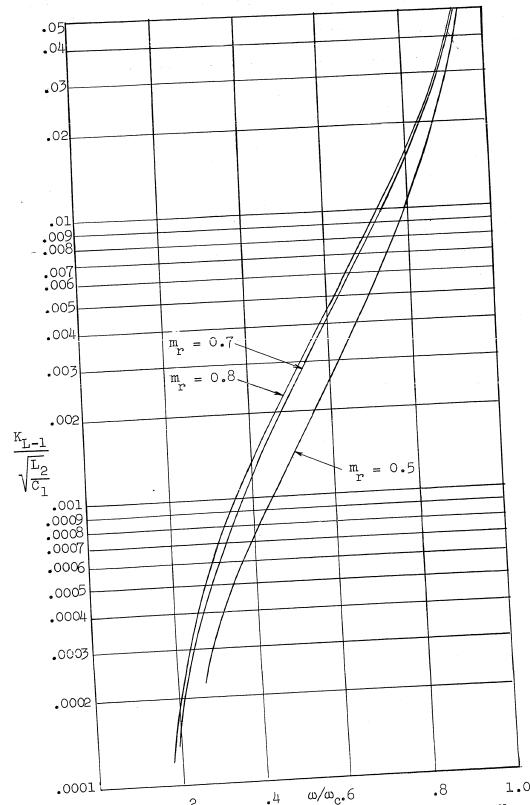


FIG. 7.82.--Longitudinal interaction impedance  $K_{L-1}$  of the four-element band-pass circuit,  $x_1 = 0.16$ .

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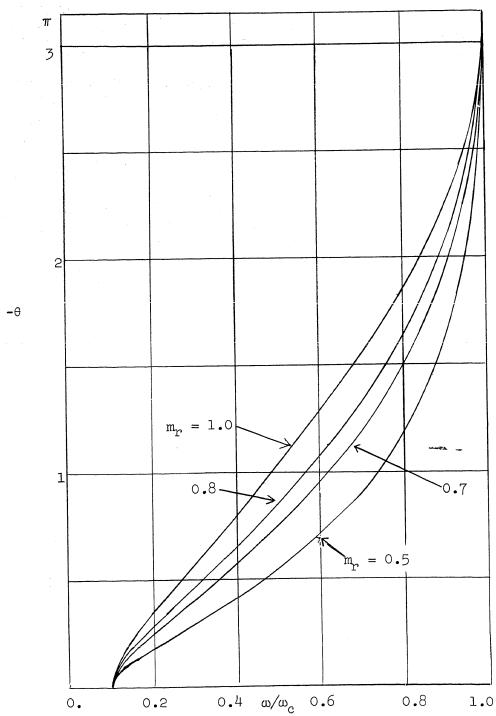


FIG. 7.83.--Phase angle per section of the four-element band-pass circuit,  $x_1 = 0.100$ .

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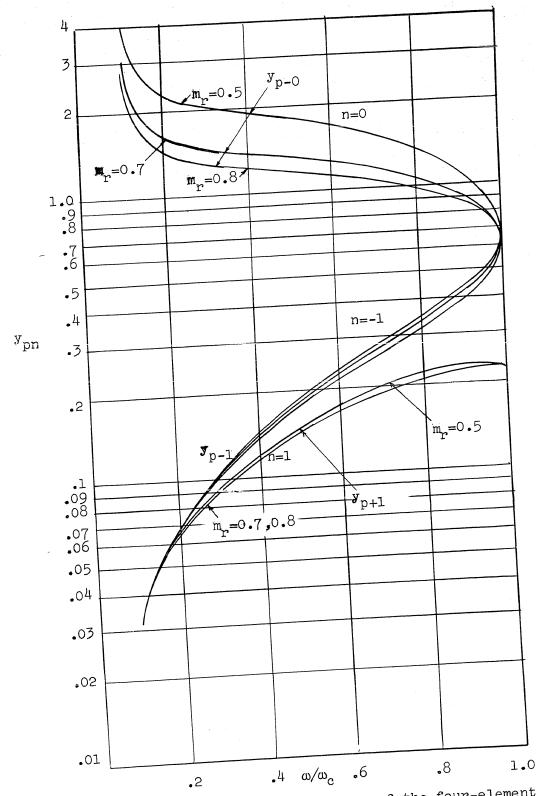


FIG. 7.84.--Phase velocity  $y_{pn}$  of the four-element band-pass circuit,  $x_1 = 0.100$ .

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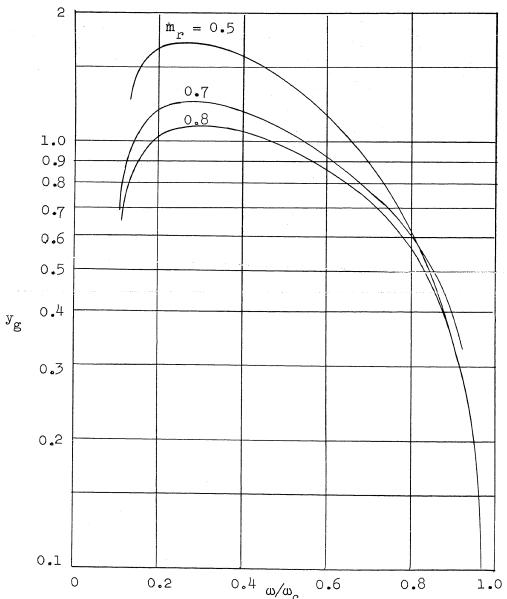


FIG. 7.85.--Group velocity  $y_g$  of the four-element band-pass circuit,  $x_1 = 0.100$ .

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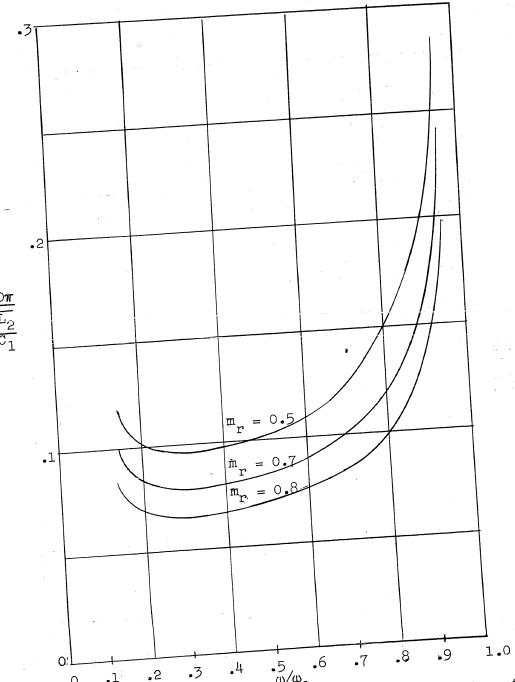


FIG. 7.86.--Circuit impedance  $Z_{0\pi}$  of the four-element band-pass circuit of PI section,  $x_1 = 0.100$ .

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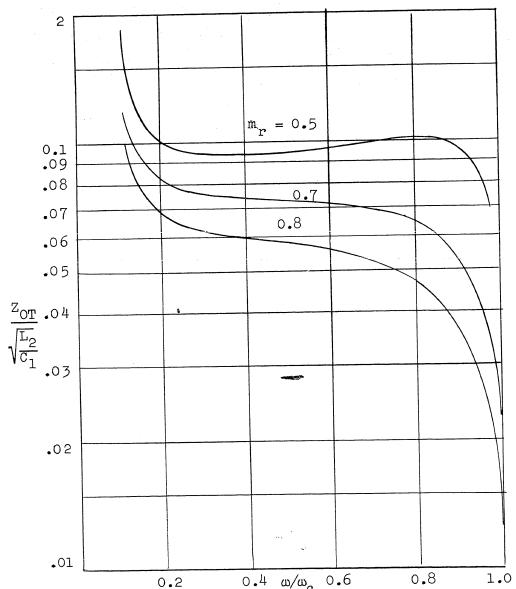


FIG. 7.87.--Circuit impedance  $Z_{OT}$  of the four-element band-pass circuit of Tee section  $x_1 = 0.100$ .

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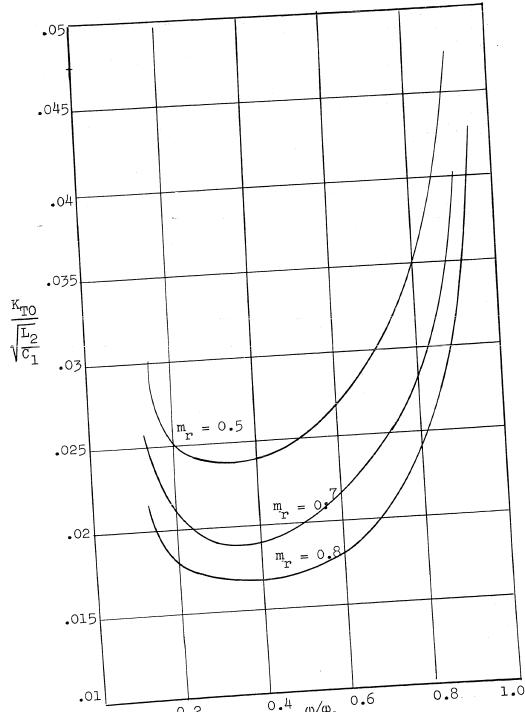


FIG. 7.88.--Transverse interaction impedance  $K_{T0}$  of the four-element band-pass circuit,  $x_1 = 0.100$ .

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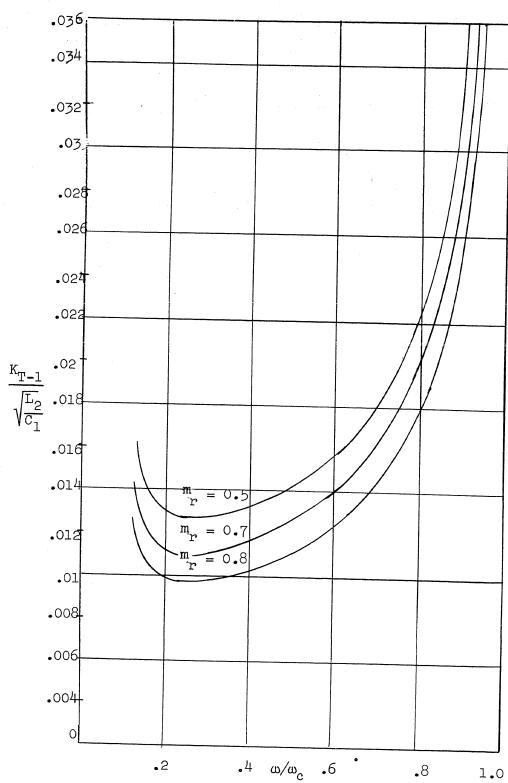


FIG. 7.89.--Transverse interaction impedance  $K_{T-1}$  of the four-element band-pass circuit,  $x_1 = 0.105^1$ .

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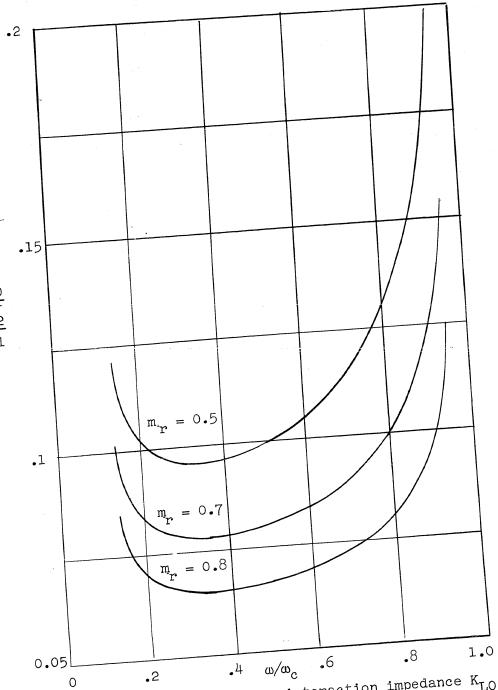


FIG. 7.90.--Longitudinal interaction impedance  $K_{L0}$  of the band-pass circuit,  $x_1 = 0.100$ .

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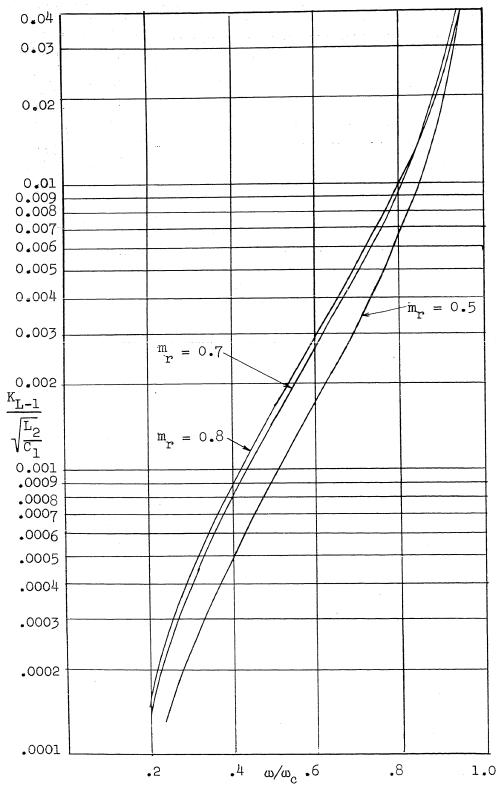


FIG. 7.91.—Longitudinal interaction impedance  $K_{L-1}$  of the four-element band-pass circuit,  $x_1=0.001$ .

## REFERENCES

1. C. T. Sah, "Analysis for external-circuit traveling-wave tubes," T.R. No. 1, Nonr 225 (24), Stanford Electronics Laboratories, Stanford University, California.

